CIS 301: Logical Foundations of Programming, Exam 2

November 16, 2001

General Notes

- Open textbook (Huth and Ryan), open class notes, open solutions of homework assignments.
- Please write your name on this page.

Good Luck!

1. *9 points*. Prove the following sequent:

$$\forall z \exists x((\forall y P(x,y)) \vee Q(x,z)) \vdash \forall y \exists x(P(x,y) \vee Q(x,y))$$

2. *8 points*. Recall the BNF of terms and formulas in predicate logic:

$$
\begin{aligned}
t &::= c \mid x \mid f(t_1, \ldots, t_n) \\
\phi &::= P(t_1, \ldots, t_n) \mid \neg\phi \mid \phi_1 \vee \phi_2 \mid \phi_1 \wedge \phi_2 \mid \phi_1 \rightarrow \phi_2 \mid \forall x\ \phi \mid \exists x\ \phi
\end{aligned}
$$

Write the function freeVar that takes any predicate logic formula ϕ as input and returns the *set* of free variables in ϕ. For example,

$$
\begin{aligned}
\mathsf{freeVar}(\forall x \forall y (P(x,y,z) \rightarrow Q(y,u))) &= \{z, u\} \\
\mathsf{freeVar}(\forall x \forall y R(x,y)) &= \emptyset
\end{aligned}
$$

3. *8 points*. Show that $\forall x(P(x) \lor Q(x)) \not\models (\forall x P(x)) \lor (\forall x Q(x))$. You should proceed in two steps.

   (a) *4 points*. Construct a model $\mathcal{M}$ such that $\mathcal{M} \models_{[\,]} \forall x(P(x) \lor Q(x))$.

   (b) *4 points*. Now show that $\mathcal{M} \not\models_{[\,]} (\forall x P(x)) \lor (\forall x Q(x))$.

   Note the empty environment attached to $\models$ above; this is because neither of the formulas contain free variables.