CIS 301: Logical Foundations of Programming, Exam 3

December 7, 2001

General Notes

- Open textbook (Huth and Ryan), open class notes, open solutions of homework assignments.
- Please write your name on this page.

Good Luck!

1. (a) *2 points.* Draw the syntax tree of the formula $\forall x(R(x,b)) \wedge \forall y((\exists z(R(y,z))) \rightarrow R(a,y))$

   (b) *2 points.* In the above formula, which variables are free and which are bound?

   (c) *5 points.* Now prove the following sequent:

$$\forall x(R(x,b)) \wedge \forall y((\exists z(R(y,z))) \rightarrow R(a,y)) \vdash \exists u \forall v(R(u,v))$$

2. *8 points*. Recall the BNF of terms and formulas in predicate logic:

$$\begin{array}{rcl} t & ::= & c \mid x \mid f(t_1, \ldots, t_n) \\ \phi & ::= & P(t_1, \ldots, t_n) \mid \neg\phi \mid \phi_1 \vee \phi_2 \mid \phi_1 \wedge \phi_2 \mid \phi_1 \rightarrow \phi_2 \mid \forall x\ \phi \mid \exists x\ \phi \end{array}$$

Write the function boundVar that takes any predicate logic formula ϕ as input and returns the *set* of bound variables in ϕ.

3. *8 points*. Consider the following program P:

```
i := 1;
while (n >= 2*i) {
  i := 2*i
}
```

(a) *4 points*. What is a suitable invariant for P?

(b) *4 points*. Use the invariant to show $\vdash_{\text{par}} \{n > 0\}\ P\ \{(0 < i \leq n) \wedge (\exists p.i = 2^p) \wedge (n < 2 * i)\}$.