CIS 301: Logical Foundations of Programming, Exam 2

November 18, 2002

General Notes

- Open textbook (Huth & Ryan), open class notes, open solutions of homework assignments.
- Please write your name on this page.

Good Luck!

NAME:

1. *6 points*. Consider the following model M:

- the universe A is the set of integers;
- the function f (arity 1) is interpreted as the successor function, that is

$$f^M(a) \quad = \quad a+1$$

- and the predicate Q (arity 1) is interpreted as the function testing whether a number is even, that is

$$\begin{array}{rcll} Q^M(a) & = & T & \text{if } a = 2b \text{ for some integer } b \\ Q^M(a) & = & F & \text{otherwise} \end{array}$$

I. Evaluate the formula $Q(f(x))$ in the model M with respect to an environment mapping x to 3, that is decide whether $M \models_{x \mapsto 3} Q(f(x))$. In other words: calculate (also writing intermediate steps) the value of

$$[\![Q(f(x))]\!]^M_{x \mapsto 3} \quad =$$

II. Find out whether it holds that

$$M \models \forall x(Q(x) \vee Q(f(x)))$$

and explain why, or why not.

2. *7 points.* Prove the following sequent

$$\exists x \forall y P(x, y) \vdash \forall y \exists x P(x, y)$$

(The rules you may use are those from Sect. 2.3.1 and p. 31 in Huth & Ryan. In particular, you cannot use the equivalences listed in Theorem 2.9, nor the "proved equivalences" given on p. 34.)

3. *7 points.* Consider the sequent you have just proven in Question 2.
Does the converse

$$\forall y \exists x P(x, y) \vdash \exists x \forall y P(x, y)$$

hold? Argue why, or why not.
(Doing so, you are allowed to exploit the fact that predicate logic is sound and complete, cf. Huth & Ryan, p.93.)

4. *5 points.*

The *factorial function* fac is defined recursively by

$$\begin{array}{rcl} \mathsf{fac}(1) & = & 1 \\ \mathsf{fac}(n+1) & = & (n+1) \cdot \mathsf{fac}(n) \end{array}$$

And as usual, the exponentiation function m^p is given by $\overbrace{m \cdot \ldots \cdot m}^{p \text{ times}}$.

Prove by induction that for all $n \geq 1$, it holds that

$$\mathsf{fac}(n) \leq n^n$$