CIS 301: Logical Foundations of Programming, Exam 3

December 18, 2002

General Notes

- This set contains four (4) questions.
- Time: 80 minutes (from 11:50am to 1:10 pm).
- Open textbook (Huth & Ryan), open class notes, open solutions of homework assignments.
- Please write your name on this page.

Good Luck!

NAME:

1. *7 points.*

Prove the sequent

$$\forall x(P(x) \lor Q(x)) \land \neg\exists x P(x) \vdash \forall x Q(x)$$

using only the rules from Fig. 1.2 (p. 31) and Sect. 2.3.1 in Huth & Ryan.

2. *6 points.*

(a) Argue why the result proved in Question 1 implies that

$$\forall x(P(x) \lor Q(x)) \land \neg \exists x P(x) \models \forall x Q(x)$$

(b) Show (by giving a model that is a counterexample) that the converse does *not* hold. That is:

$$\forall x Q(x) \not\models \forall x(P(x) \lor Q(x)) \land \neg \exists x P(x)$$

3. *4 points.*

Recall that the *fibonacci function* fib is recursively defined by

$$\begin{aligned}
\mathsf{fib}(1) &= 1 \\
\mathsf{fib}(2) &= 1 \\
\mathsf{fib}(n+2) &= \mathsf{fib}(n+1) + \mathsf{fib}(n) \qquad \text{for } n \geq 1
\end{aligned}$$

Prove by induction that for all $n \geq 1$ it holds that

$$\mathsf{fib}(n) < 2^n$$

4. *8 points.*

With P the program below and with fib as defined in Question 3, prove that

$$\vdash_{\text{par}} (\!|\ x \geq 2\ |\!)\ \ P\ \ (\!|\ y = \mathsf{fib}(x)\ |\!)$$

by annotating P so as to create a proof tableau (cf. p. 237 in Huth & Ryan). This includes coming up with a suitable invariant for the `while` loop.

You can ignore the implicit demand that the value of `x` should not change (that is, don't bother about introducing a logical variable x_0).

```
a = 2 ;

y = 1 ;

w = 1 ;

while a != x {

   a = a + 1 ;

   h = y ;

   y = y + w ;

   w = h

}
```