The Validity and Soundness of Arguments

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Introduction

Atomic Sentences (summary)

Logical Consequence

Demonstrating Non-consequence

Road Map

Two main aims of book (p.2):

1. help you learn language of first-order logic (FOL)
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- ▶ Chapter 1 takes the first step of (1)
- ▶ Chapter 2 takes the first step of (2)

Atomic Sentences

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	functions	predicates
result is	object	truth value
spelling is	lower case	capitalized
can be nested?	yes	no

Example Worlds

	constants	functions		predicates	
		arity 1	arity 2	arity 1	arity 2
Arithmetic	0,1,2,...	sin, cos	$+,-$		$<$
Family	max, claire	father		Pet	Older
Tarski's World	a, b, ...			Cube	LeftOf

- ▶ many functions and predicates with arity 2 are written infix: $x + y$, $x < y$, $x = y$
- ▶ functions can be added to Tarski's world (p.33, and homework exercises 1.13 & 1.14)
- ▶ The identity predicate "$=$" is relevant in all worlds!

Motivation

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Fitch format

> a is larger than b
> b is larger than c
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Valid and Sound Arguments

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- ▶ valid: it is not possible for the conclusion to be false if the premises are true.

Valid and Sound Arguments

Socrates is a man   true (history)
All men are mortal   true (biology)
Socrates is mortal

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- ▶ sound: it is valid, and its premises are true.

Valid and Sound Arguments

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Socrates is mortal	true (history: hemlock, 399 BC)

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- ▶ sound: it is valid, and its premises are true.
  (so also its conclusion is true)

Unsound Arguments

Scruffy is a man
All men are mortal
Scruffy is mortal

This argument is

► valid, as same structure as the previous argument

Unsound Arguments

Scruffy is a man	false
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Red Sox win the World Series each year	false
Red Sox will win the 2004 World Series	?

This is also valid, but, alas, not sound.

Invalid Arguments

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Socrates is a man

This argument has a different structure than what we have seen, and is invalid.

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All men are mortal	true
Socrates is a man	false

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Counterexample: Socrates might be a dog

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To decide whether an argument is

▶ valid: it is sufficient to examine the structure of the argument

▶ sound: we must examine history, biology, baseball, etc.

Therefore the focus of logic, and this course, is on validity of argument, rather than on soundness.

Counterexamples (Section 2.5)

Given a purported argument, a counterexample is

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Atta cannot have met with Iraqi agents in Prague on April 8, 2001
                    Atta took an airplane. . .

Submitting Counterexamples

In homeworks, you'll often be given an argument and asked to submit a world that serves as a counterexample.

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SameSize(b,c)
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b = c
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Counterexample: none, as argument is valid