# The Validity and Soundness of Arguments

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Introduction

Atomic Sentences (summary)

Logical Consequence

Demonstrating Non-consequence

# Road Map

Two main aims of book (p.2):

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- Chapter 1 takes the first step of (1)
- Chapter 2 takes the first step of (2)

# Atomic Sentences

A term $t$ is built from constants and function symbols:

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| | functions | predicates |
|---|---|---|
| result is | object | truth value |
| spelling is | lower case | capitalized |
| can be nested? | yes | no |

# Example Worlds

| | constants | functions | | predicates | |
|---|---|---|---|---|---|
| | | arity 1 | arity 2 | arity 1 | arity 2 |
| Arithmetic | 0,1,2,... | sin, cos | +,- | | $<$ |
| Family | `max`, `claire` | `father` | | `Pet` | `Older` |
| Tarski's World | `a`, `b`, ... | | | `Cube` | `LeftOf` |

- many functions and predicates with arity 2 are written infix: $x + y$, $x < y$, $x = y$
- functions can be added to Tarski's world (p.33, and homework exercises 1.13 & 1.14)
- The identity predicate "=" is relevant in all worlds!

# Motivation

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| | b is larger than c |
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## Fitch format

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# Valid and Sound Arguments

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All men are mortal
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∴ Socrates is mortal

This classical argument is

- valid: it is not possible for the conclusion to be false if the premises are true.

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| Socrates is a man | true (history) |
| All men are mortal | true (biology) |
| Socrates is mortal | |

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# Valid and Sound Arguments

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| All men are mortal | true (biology) |
| Socrates is mortal | true (history: hemlock, 399 BC) |

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- sound: it is valid, and its premises are true.
(so also its conclusion is true)

# Unsound Arguments

Scruffy is a man
All men are mortal
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Scruffy is mortal

This argument is

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# Unsound Arguments

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| Scruffy is a man | false |
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- unsound, since Scruffy is a cat
  (conclusion happens to be true)

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| Red Sox win the World Series each year | false |
| Red Sox will win the 2004 World Series | ? |

This is also valid, but, alas, not sound.

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To decide whether an argument is

- valid: it is sufficient to examine the structure of the argument
- sound: we must examine history, biology, baseball, etc.

Therefore the focus of logic, and this course, is on validity of argument, rather than on soundness.

# Counterexamples (Section 2.5)

Given a purported argument, a counterexample is

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- enough to show that the argument is invalid: the conclusion does *not* follow from the premises (is non sequitur).

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Atta took an airplane. . .

# Submitting Counterexamples

In homeworks, you'll often be given an argument and asked to submit a world that serves as a counterexample.

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Counterexample: none, as argument is valid