CIS 301: Logical Foundations of Programming, Exam II

April 11, 2008, 10:30-11:20am

General Notes

- Open textbook (Barwise & Etchemendy), open class notes, open solutions of homework assignments. No use of laptops or other computing devices.
- Please write your name on this page.

Good Luck!

NAME:

1. *5 points.* Translate the following English sentences into (the Tarski's World variant of) FOL. To earn full points, your sentences should be such that negation applies to atomic sentences only.

No tetrahedron is small

There is exactly one large dodecahedron

For each cube there is some other object of the same size

2. *5 points.* Below is written a program that multiplies the integer `x` by 4 and stores the result in `y`. Your task is to give a formal correctness proof (cf. Section 7 of the notes) that establishes the postcondition $\mathtt{y} = 4 \cdot \mathtt{x}$. That is, you must add assertions, and list the inference rules used to derive each assertion (you do not need to justify applications of **Implies**).

Hint: The loop invariant is that $\mathtt{y} = 4 \cdot (\mathtt{x} - \mathtt{w})$.

```
y := 0;

w := x;

while w ≠ 0 do

    y := y + 4;

    w := w − 1

od
```

Notice that so far, we have not stated what the precondition ϕ_0 is.

- What must ϕ_0 satisfy in order for the above proof to work out?

- Can you think of anything else that ϕ_0 must satisfy?

3. *5 points.* For the following argument, the conclusion is a FO consequence of the premises. Therefore the argument has a proof in system F.

$$\begin{array}{|l} \forall x(P(x) \to (Q(x) \lor R(x))) \\ \exists y(P(y) \land \neg Q(y)) \\ \hline \exists w(P(w) \land R(w)) \end{array}$$

Your task is to write such a proof, where you annotate each step with the rule you have used to derive that step, and also cite the supporting sentences. The rules you may use are the introduction and elimination rules for the quantifiers, **Taut Con**, and the introduction and elimination rules for the boolean connectives.

4. *5 points.* Consider the English sentences (the first translated into FOL):

(a) *all cubes are in the same row and in the same column*
$\forall x \forall y ((\mathbf{Cube}(x) \land \mathbf{Cube}(y)) \rightarrow (\mathbf{SameRow}(x, y) \land \mathbf{SameCol}(x, y)))$

(b) *there exists at most one cube*

Give a FO-counterexample (cf. p. 270) that shows that (b) is *not* a FO-consequence of (a).

Give an axiom such that (b) *is* a FO consequence of (a) and that axiom.

Give a FO-counterexample that shows (a) is *not* a FO-consequence of (b).

Give an axiom such that (a) *is* a FO consequence of (b) and that axiom.