CIS 301, Spring 2008, Exam II, model solutions

Question 1

$\forall \mathbf{x}(\mathbf{Tet}(\mathbf{x}) \rightarrow \neg \mathbf{Small}(\mathbf{x}))$

$\exists \mathbf{x}(\mathbf{Large}(\mathbf{x}) \wedge \mathbf{Dodec}(\mathbf{x}) \wedge \forall \mathbf{y}((\mathbf{Large}(\mathbf{y}) \wedge \mathbf{Dodec}(\mathbf{y})) \rightarrow \mathbf{y} = \mathbf{x}))$

$\forall \mathbf{x}(\mathbf{Cube}(\mathbf{x}) \rightarrow \exists \mathbf{y}(\mathbf{y} \neq \mathbf{x} \wedge \mathbf{SameSize}(\mathbf{x}, \mathbf{y})))$

Question 2

$\{0 = 4 \cdot (\mathtt{x} - \mathtt{x})\}$	
$\mathtt{y} := 0;$	
$\{\mathtt{y} = 4 \cdot (\mathtt{x} - \mathtt{x})\}$	**Assignment**
$\mathtt{w} := \mathtt{x};$	
$\{\mathtt{y} = 4 \cdot (\mathtt{x} - \mathtt{w})\}$	**Assignment**
while $\mathtt{w} \neq 0$ do	
$\{\mathtt{y} = 4 \cdot (\mathtt{x} - \mathtt{w}) \wedge \mathtt{w} \neq 0\}$	**WhileTrue**
$\{\mathtt{y} + 4 = 4 \cdot (\mathtt{x} - (\mathtt{w} - 1))\}$	**Implies**
$\mathtt{y} := \mathtt{y} + 4;$	
$\{\mathtt{y} = 4 \cdot (\mathtt{x} - (\mathtt{w} - 1))\}$	**Assignment**
$\mathtt{w} := \mathtt{w} - 1$	
$\{\mathtt{y} = 4 \cdot (\mathtt{x} - \mathtt{w})\}$	**Assignment**
od	
$\{\mathtt{y} = 4 \cdot (\mathtt{x} - \mathtt{w}) \wedge \mathtt{w} = 0\}$	**WhileFalse**
$\{\mathtt{y} = 4 \cdot \mathtt{x}\}$	**Implies**

Since $0 = 4 \cdot (\mathtt{x} - \mathtt{x})$ always holds, ϕ_0 could be anything. On the other hand, remember that the above proof shows only *partial* correctness. To ensure termination, we must demand that $\mathtt{x} \geq 0$.

Question 3

1. $\forall x(P(x) \rightarrow (Q(x) \vee R(x)))$	
2. $\exists y(P(y) \wedge \neg Q(y))$	
3. $\boxed{a}$ $P(a) \wedge \neg Q(a)$	(name the object satisfying P but not Q)
4. $P(a) \rightarrow (Q(a) \vee R(a))$	$\forall$ **Elim** : 1
5. $P(a) \wedge R(a)$	**TautCon** : 3, 4
6. $\exists w(P(w) \wedge R(w))$	$\exists$ **Intro** : 5
7. $\exists w(P(w) \wedge R(w))$	$\exists$ **Elim** : 2, 3–6

Question 4

If we replace **SameRow** and **SameCol** by **SameSize** then (a) says that all cubes have the same size — which clearly does not imply (b). To make (b) a FO consequence of (a), we must capture that two different objects cannot occupy the same square; this can be achieved by the axiom

$$\forall x \forall y((\mathbf{SameRow}(x, y) \wedge \mathbf{SameCol}(x, y)) \rightarrow x = y).$$

If we replace **SameRow** and **SameCol** by **Larger** then (a) says that all cubes are larger than each other, which can only hold if there are no cubes — a stronger property than (b), and hence not implied by (b). To make (a) a FO consequence of (b), we must capture that a cube is in the same row and same column as itself; this can be achieved by the axiom (where of course we could omit the antecedent "**Cube**(x)")

$$\forall x(\mathbf{Cube}(x) \rightarrow (\mathbf{SameRow}(x, x) \wedge \mathbf{SameCol}(x, x))).$$