CIS 761. Database Management Systems
Lecture notes on "The Chase"

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A typical problem

Assume that the relation r satisfies the functional dependency

$$A \rightarrow B$$

and the multi-valued dependency

$$B \rightarrow\rightarrow C.$$

We want to use the above dependencies to *simplify* the query q given by

$$q = \Pi_{ABD}(r) \bowtie \Pi_{AC}(r).$$

The solution

Assume that t is a tuple in q. Let $t = (a, b, c, d)$, then we know that $(a, b, d) \in \Pi_{ABD}(r)$ and that $(a, c) \in \Pi_{AC}(r)$. Therefore, there exists c', b', d' such that r contains the tuples

A	B	C	D
a	b	c'	d
a	b'	c	d'

Since r satisfies $A \rightarrow B$, we infer that $b = b'$, and the situation is therefore that r contains the tuples

A	B	C	D
a	b	c'	d
a	b	c	d'

and since r satisfies $B \twoheadrightarrow C$, r also contains the tuples

A	B	C	D
a	b	c'	d
a	b	c	d'
a	b	c	d
a	b	c'	d'

In particular, we see that $t = (a, b, c, d) \in r$. Since t was an arbitrary tuple in q, this shows that $q \subseteq r$. Clearly, $r \subseteq q$, so $q = r$. Thus, the complex query q can be reduced to the simple query r.

For more material on "the chase", see [Abiteboul & Hull & Vianu, 1995].