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Neural-Symbolic Integration

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St. Andrews, Scotland, January 2009

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- Diplom (Mathematics) Univ. of Tübingen 1998
- PhD (Mathematics), Nat. Univ. of Ireland Cork 2001
- 2001-2004 AI Institute TU Dresden
- 2005 Habilitation (Computer Science)
- since 2004 Assistant Professor, AIFB, Univ. of Karlsruhe

  – Knowledge Representation and Reasoning for the **Semantic Web**
  – Neural-Symbolic Integration
  – Mathematical Foundations of Artificial Intelligence

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Main references for this talk

- S. Bader, P. Hitzler, S. Hölldobler. Connectionist Model Generation: A First-Order Approach. **Neurocomputing** 71, 2008, 2420-2432.
- S. Bader, P. Hitzler, S. Hölldobler, A. Witzel. A Fully Connectionist Model Generator for Covered First-Order Logic Programs. In: Manuela M. Veloso, Proceedings of the Twentieth International Joint Conference on Artificial Intelligence, **IJCAI-07**, Hyderabad, India, January 2007, AAAI Press, Menlo Park CA, 2007, pp. 666-671.
- P. Hitzler, S. Hölldobler and A. K. Seda. Logic Programs and Connectionist Networks. **Journal of Applied Logic** 2(3), 2004, 245-272.

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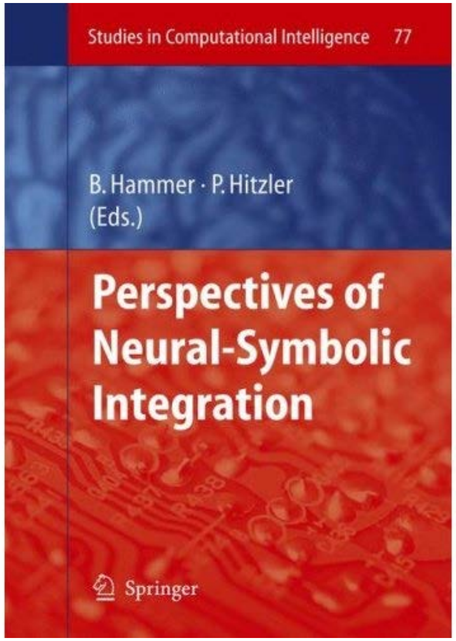
State-of-the-art collection:



Barbara Hammer, Pascal Hitzler (eds.)

Perspectives of Neural-Symbolic Integration.

Studies in Computational Intelligence 77.

Springer, 2007.

With contributions by

Barreto, de Raedt, Frasconi, Garcez, Gust
Hölldobler, **Komendantskaya**, Kühnberger, Ritter,
Saunders, Seda, Shastri, Sperduti, Tino

5th International Workshop on Neural-Symbolic Learning and Reasoning

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http://www.neural-symbolic.org

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Artur d'Avila Garcez, City University London, UK

Pascal Hitzler, University of Karlsruhe (TH), Germany

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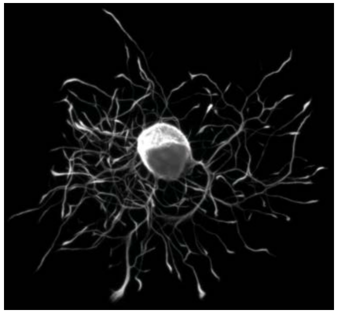
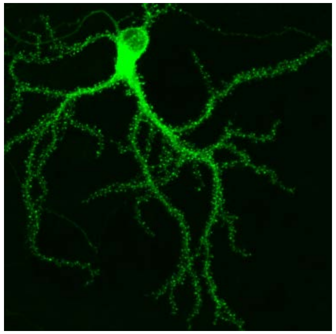
Neural-symbolic Integration





1. **Why neural-symbolic integration?**
2. Earlier work
3. The neural-symbolic learning cycle
4. Propositional fixation
5. The cycle for first-order logic
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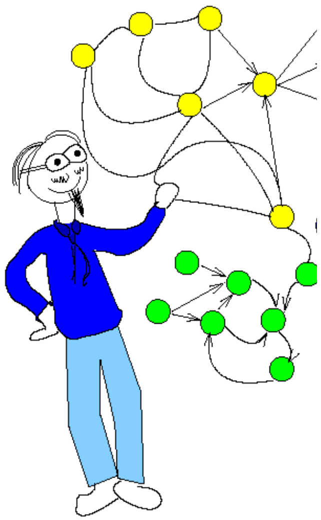
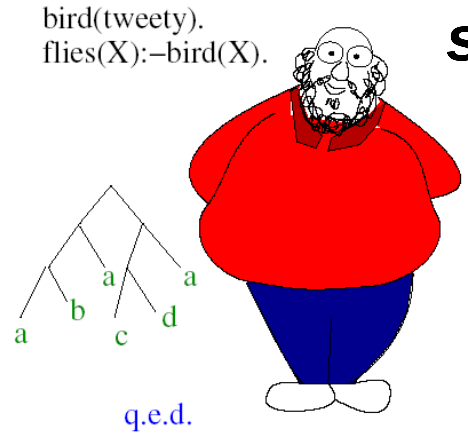
Why neural-symbolic integration?

Neural-symbolic Integration

connectionism





symbolic AI

- Artificial neural networks and symbolic AI are two fundamentally different paradigms in AI.
- Their strengths and weaknesses are complementary.
- *Neural-symbolic Integration* is about integrating the paradigms while retaining their strengths.

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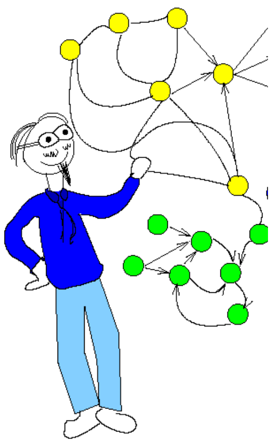
Artificial neural networks



- Powerful machine-learning paradigm.
- Inspired by Biology/Neuroscience. ☺
- Learning from noisy data possible.
- Robust. *Graceful degradation*.

- No declarative semantics. *Black boxes*. ☹
- Recursive structures difficult.
- Cannot learn with background knowledge.

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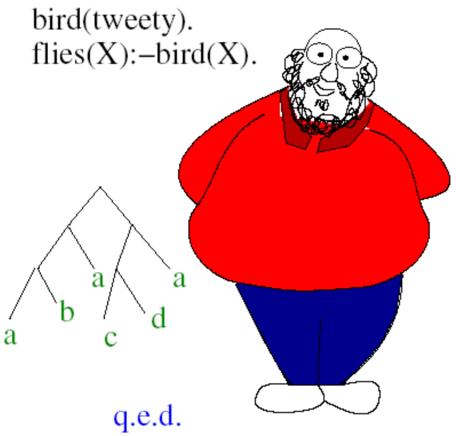
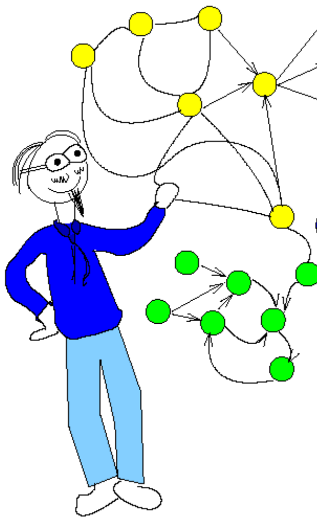
Knowledge representation/symbolic AI

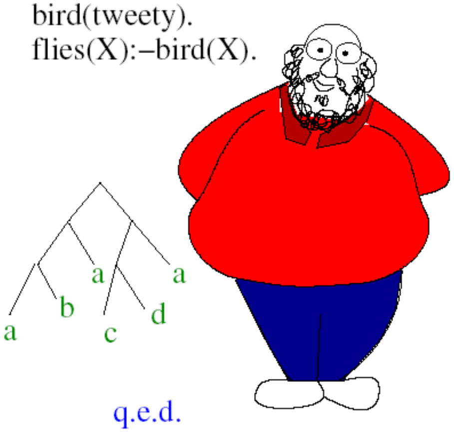


- Logic-based. *Declarative*.
- ☺ Modelled from human thinking.
- Explicit coding of knowledge.
- Highly recursive.

☹
- Learning is difficult.
- Hardly tolerant against noise.
- Reasoning has high computational complexity.

neural - ***symbolic***





☺ ☺

☹ ☹

realising connectionist processing of symbolic knowledge

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The four main problems of neural-symbolic integration

- Connectionist representation of symbolic knowledge.
- Extraction of symbolic knowledge from artificial neural networks.
- Connectionist learning of symbolic knowledge.
- Learning under background knowledge.

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Besides ...

... the *technical* motivation just given:

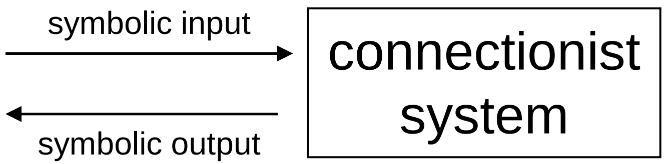
- neural-symbolic integration is about the study – from a computer science perspective – how knowledge can be processed within models of the brain
- standard artificial neural networks appear to be insufficient to capture human knowledge processing
- logic also appears to be insufficient to capture human knowledge processing

Driving motivation

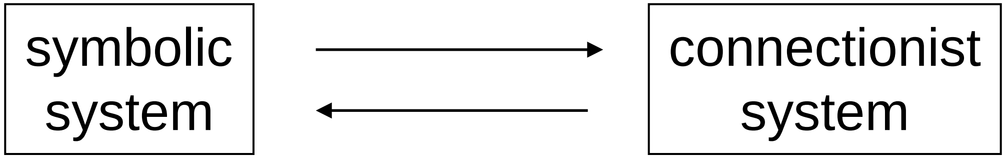
- Our approach is mainly *computer-science-driven*.
  - realisation of intelligent systems
- It contributes only indirectly to the question, how humans model reality and think about it.
- At hindsight, our approach probably rather shows, how humans do ***not*** model reality and think about it.
- Generally, neural-symbolic research requires more input from recent developments in neuroscience!

Hybrid vs. Integrated Approach

integrated



hybrid



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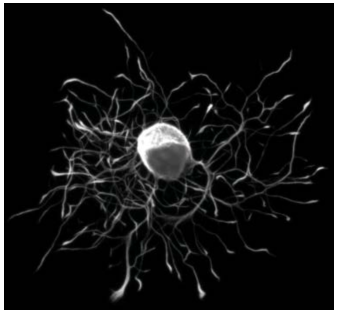
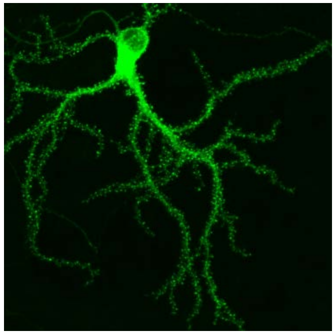
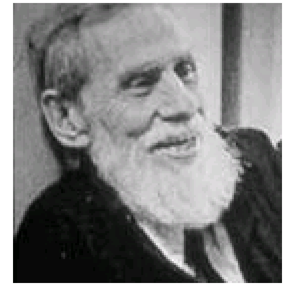




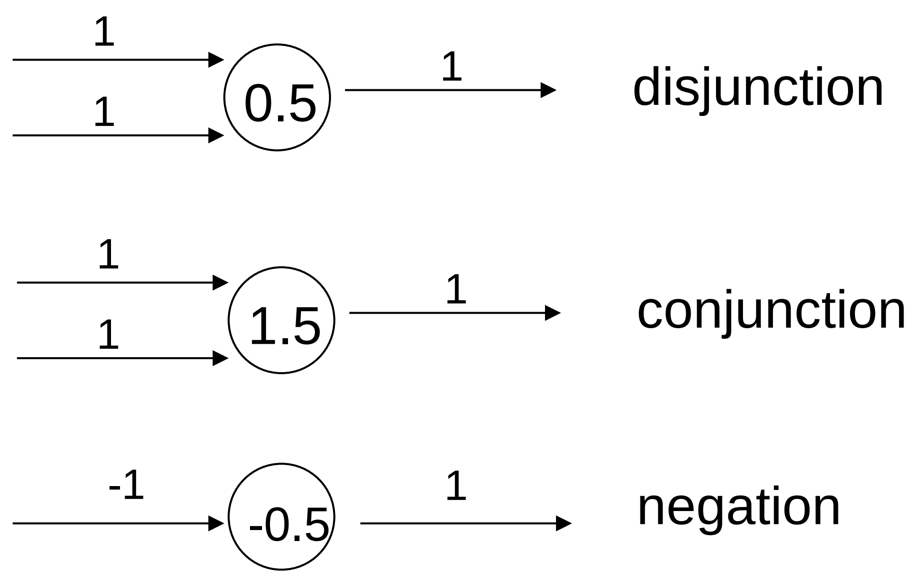
Earlier work



- McCulloch & Pitts 1943
  - Neurons with binary activation functions.
  - Modelling of propositional connectives.
  - Networks equivalent to finite automata.

Values 0 („false“) and 1 („true“) being propagated.

Simultaneous update of all nodes in network.

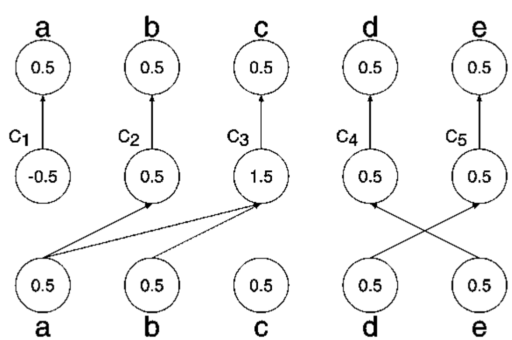


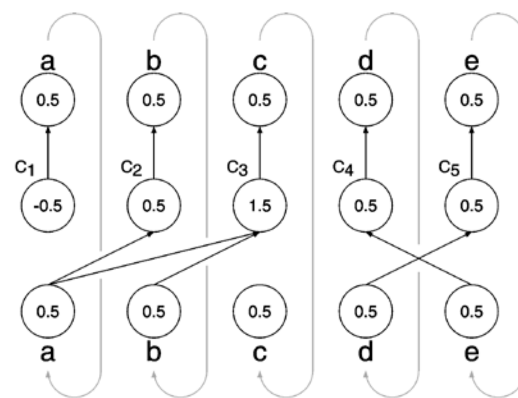
The propositional *Core Method*

- Hölldobler & Kalinke 1994
  - Extends the approach by McCulloch & Pitts.
  - Representation of propositional logic programs and their semantics.
  - „Massively parallel reasoning.“

logic program ⟶ core net ⟶ recurrent net

a Ã
b Ã a
c Ã a Æ b
d Ã e
e Ã d



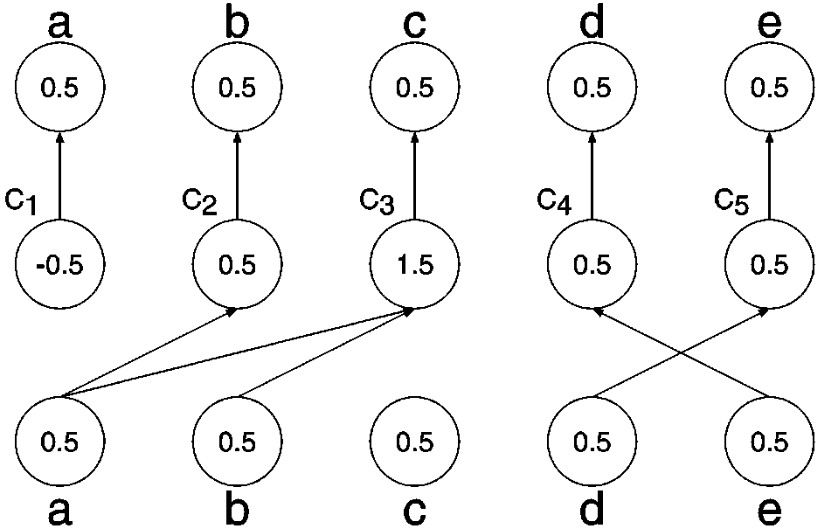


The propositional *Core Method*

Logic program P ⟶ core net

a Ã
b Ã a
c Ã a Æ b
d Ã e
e Ã d

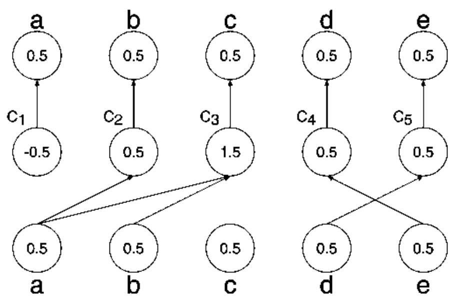
⟶

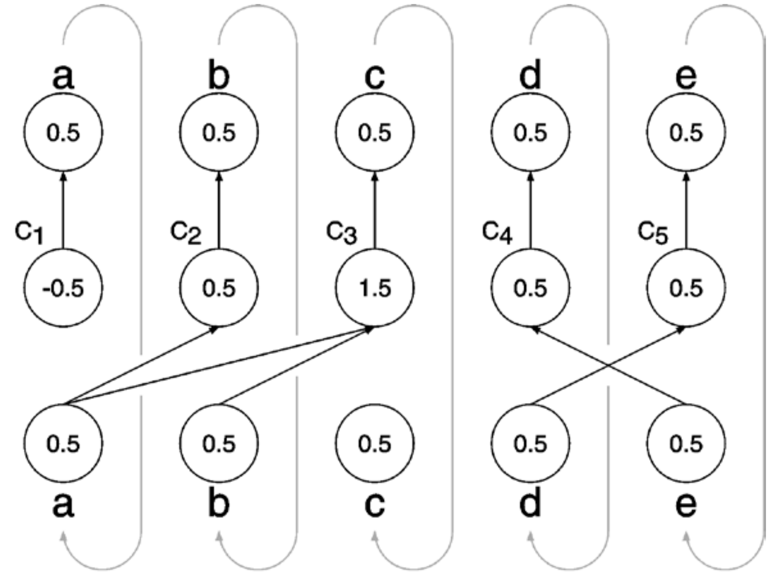


- Update „along implication“.
- Corresponds to computing the semantic operator T_P.
- T_P represents meaning (semantics) of P through its fixed points.

The propositional *Core Method*

core net → recurrent net





- Repeated updates along layers corresponds to iterations of the semantic operator.
- Semantics of the program (= fixed point of the operator) can be computed in a parallel manner.

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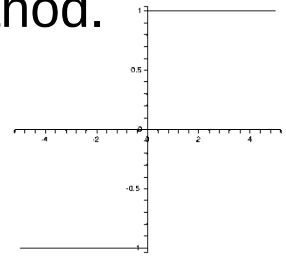
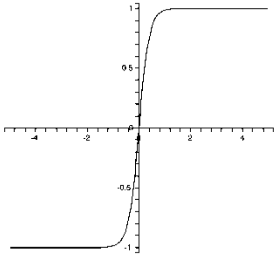
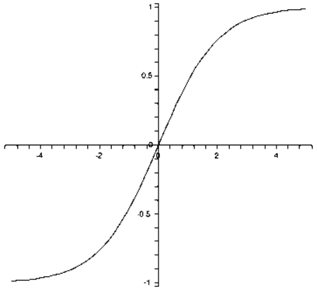
CILP – Connectionist Inductive Logic Programming

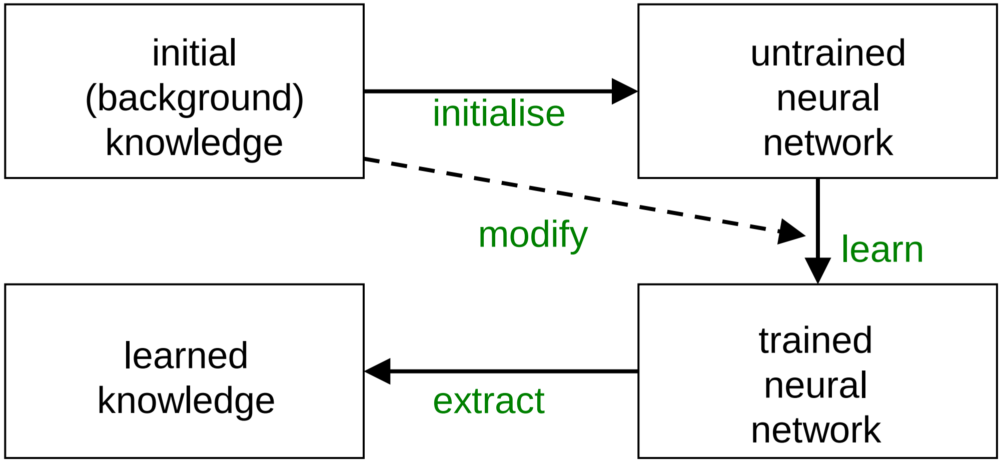
- Garcez & Zaverucha 1999
  Garcez, Broda & Gabbay 2001
- Development of a learning paradigm from the Core Method.
- Required: differentiable activation function.
  – Allows learning with standard methods.
  – Backpropagation algorithm.







- Establishing the *neural-symbolic learning cycle*.



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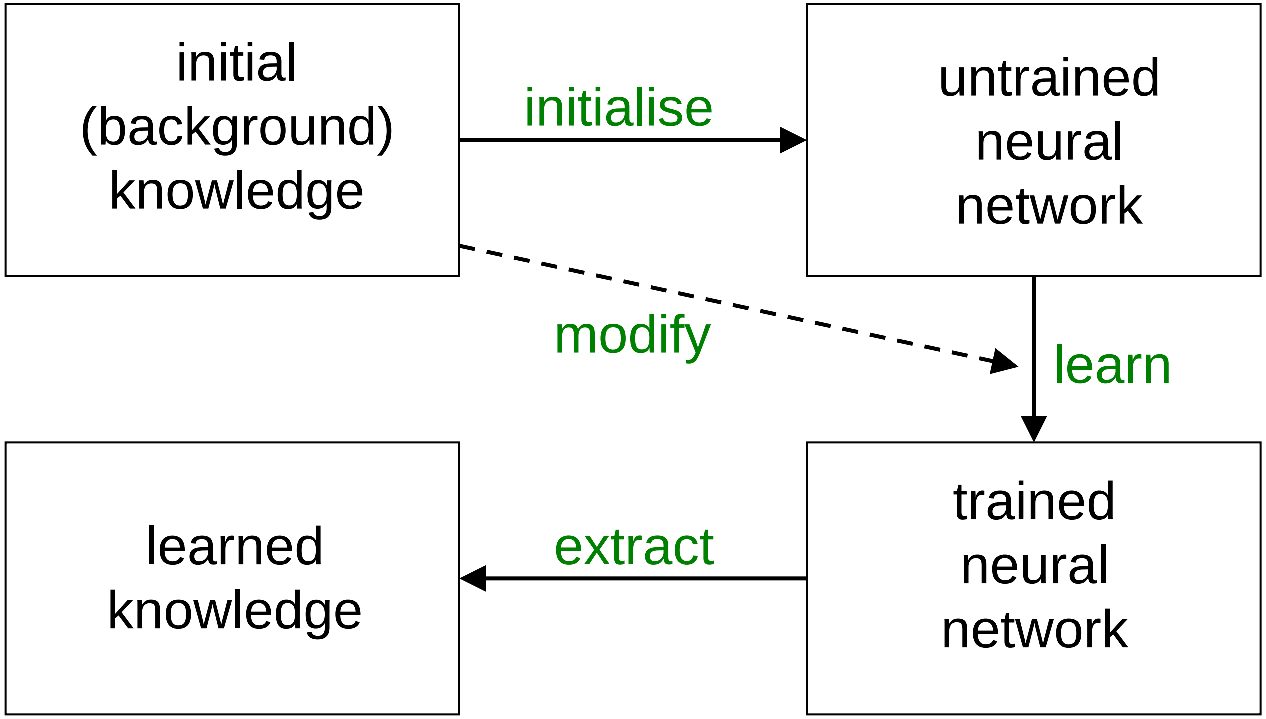
The neural-symbolic learning cycle



The four main problems of Neural-symbolic Integration.

Multi-valued Logic Programs


- Approach can be generalised to logic programs under multiple truth values.

- Rather general results can be obtained.

- No practical evaluation yet.

- [Komendantskaya, Lane, Seda, 2007]

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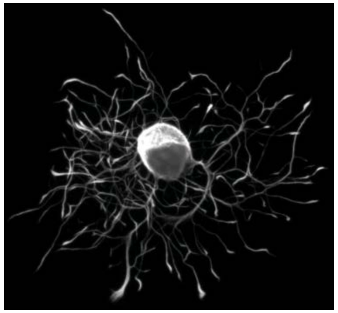
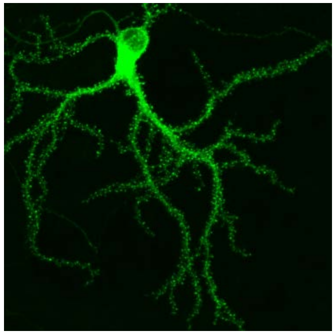


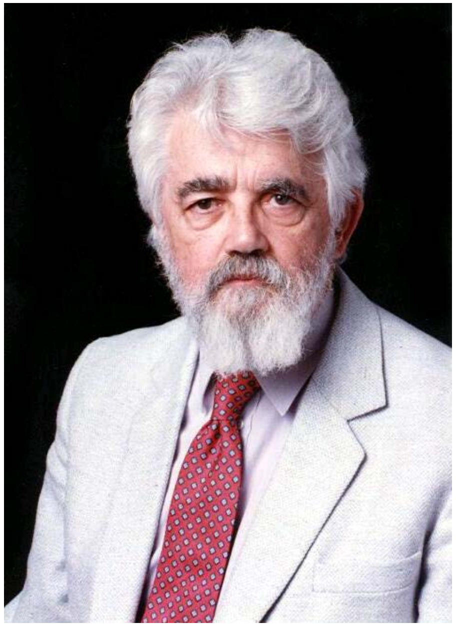


Conectionism and first-order predicate logic (PL)

- Connectionist representation of PL-knowledge very hard to realise.
  McCarthy 1988: „Propositional fixation.“

  We need to capture the infinite in a finite way.

  – infinite ground instantiations
    (8x) male(x) Æ hasSon(x,son(x)) ! father(x)
  – term representations
    member(X, [a,b,c | [d,e]])
  – variable bindings
    male(x) Æ hasSon(x,y) ! father(x)



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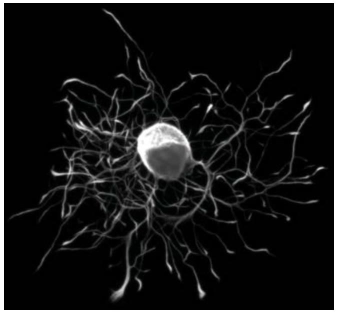
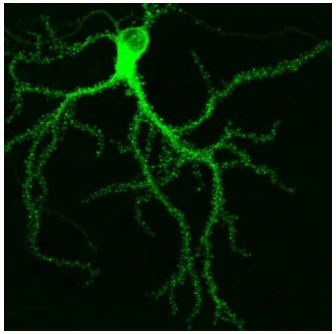


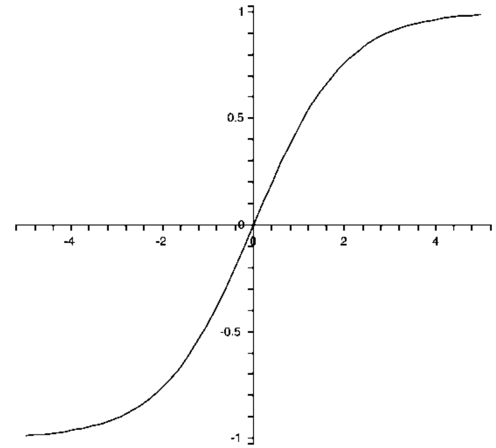


PL Core Method

- Hölldobler, Kalinke, Störr 1999
  Hitzler, Hölldobler, Seda 2004

- Idea:
  – Use results by Funahashi 1989: „Every continuous function on the reals is approximable by standard feedforward networks. "
  – Hence: Consider logic programs for which T_P-operator is continuous in this sense.

Funahashi 1989 (simplified)



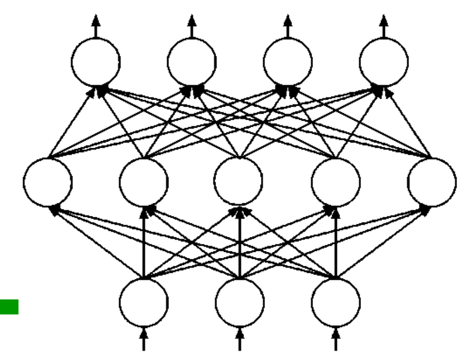
- σ sigmoidal activation function
- K µ R compact
- f: K ! R continuous
- ε > 0

Then there exists a three-layer feedforward network with activation function σ and I/O-function F, so that

$$\max_{x \in \mathsf{K}} \{ d(\mathsf{f}(x), \mathsf{F}(x)) \} < \varepsilon.$$

Here d is a metric which induces the natural topology on R.

I.e. continuous functions can be *uniformly approximated* by such networks with arbitrary accuracy.



Continuity of T_P – I

- Hitzler, Hölldobler, Seda 2004

Let $\mathbf{B}_A$ be the set of all body atoms in ground instantiated clauses of P with head A.

T_P: I_P ! I_P is called *locally finite*, if
for all atoms A and all I 2 I_P
there exists a finite S µ $\mathbf{B}_A$,
such that T_P(J)(A)=T_P(I)(A)
for all J 2 I_P which coincide with I on S.

p(s(x)) Ã p(x).
p(0)
p(x) Ã p(s(x)).

e.g. $\mathbf{B}_{p(s(0))}$ = {p(0),p(s(s(0)))}

Continuity of T_P – II

T_P: I_P ! I_P is locally finite

iff

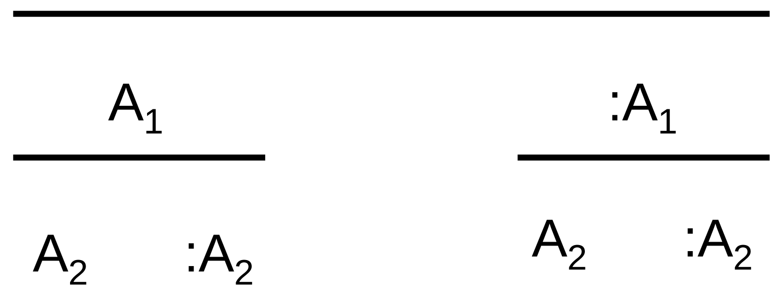
T_P is continuous in Cantor space.

- Cantor-continuity is continuity wrt. the Cantor topology on the Cantor set.
- The Cantor topology is homeomorphic to the prefix-distance on (infinite) binary trees.
- The Cantor topology is homeomorphic to the subspace topology which is induced on a subset of $\mathbb{R}$ which is compact, totally disconnected and dense in itself.

Continuity of T_P – III

- There are (uncontably) many homeomorphisms which map I_P with the Cantor topology into suitable subsets of $\mathbb{R}$.
- Locally finiteness is a logical (topology-free) characterisation of logic programs which can be represented in a a connectionist way in the sense of Funahashi.
- Problem: this argumentation is not constructive!

A_1, A_2, ... enumeration of
Herbrand base
Elements of Cantor Set
identifiable with
interpretations



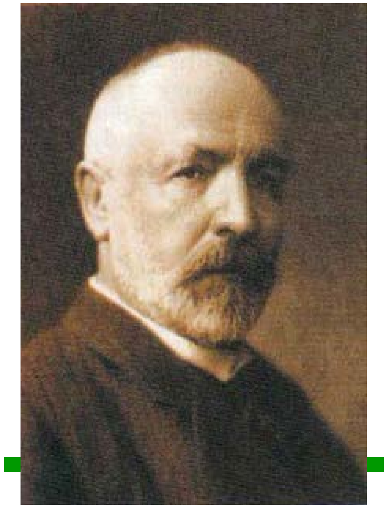
Relationship of I_P to Cantor Space

Interpretations with Cantor topology

Homeomorphism

Cantor space as compact subspace of $\mathbb{R}$

$$\begin{tikzcd} (I_P,Q) \arrow[r, "T_P"] \arrow[d, "\iota"] & (I_P,Q) \arrow[d, "\iota"] \\ \text{Cantor} \arrow[r, "\iota(T_P)"] & \text{Cantor} \end{tikzcd}$$



Georg Cantor

The Cantor topology as a paradigm bridge

- Connectionist side:
  - Cantor topology is a subtopology of the usual topology on the real numbers

- Logic Programming side:
  - Cantor topology captures useful notions of convergence of semantic operators, e.g. If T_P^n ! I (for $n \to 1$), then I is a model of P.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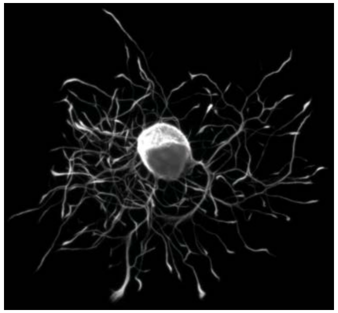
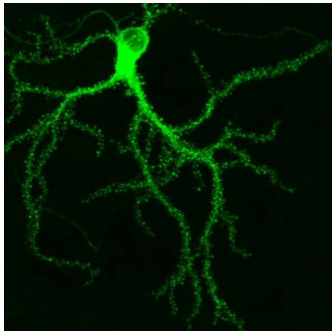
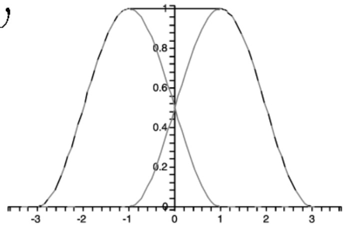




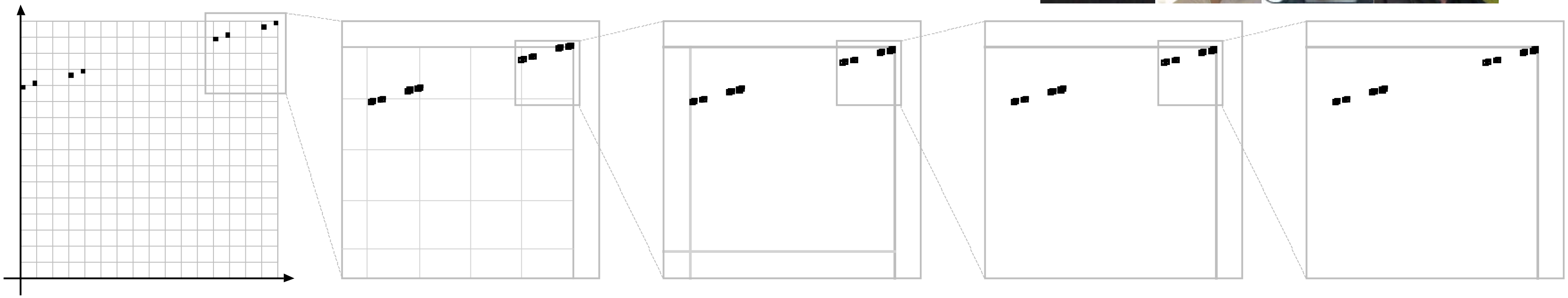
Realising the cycle: Representation of symbolic knowledge

- Bader, Hitzler, Hölldobler, Witzel – IJCAI-07
  - Algorithm for the approximate construction of neural networks from logic programs.
  - Realised for
    - RBS nets with triangular activation function
    - RBF nets with raised cosine activation function

$$\tau_{w,h,m}(x) = \begin{cases} \frac{h}{2} \cdot \left(1 + \cos\left(\frac{\pi(x-m)}{w}\right)\right) & \text{if } |x-m| < w \\ 0 & \text{otherwise} \end{cases}$$

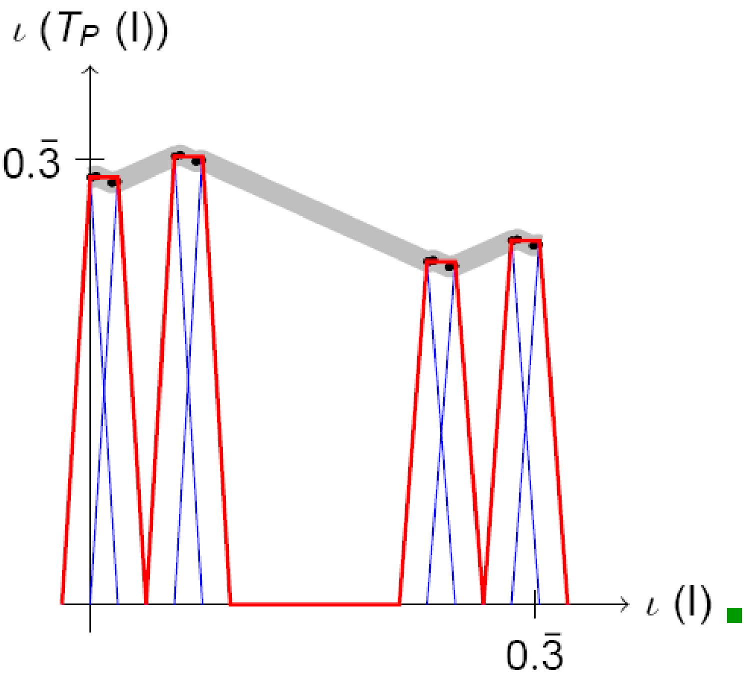


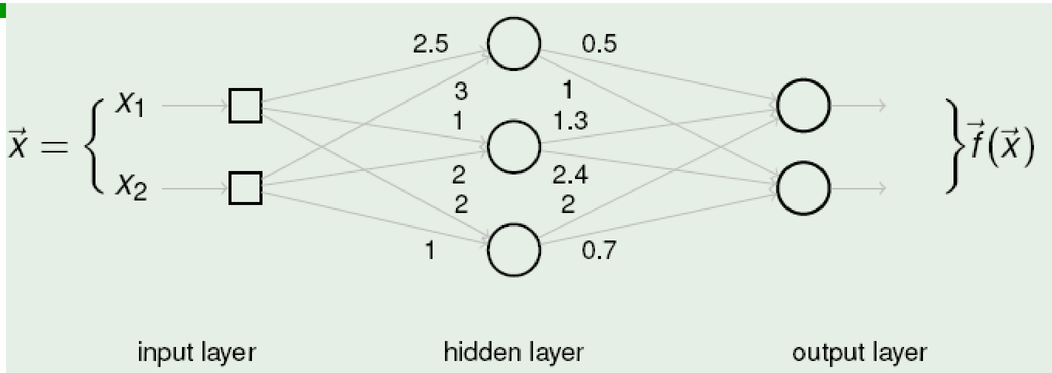
Realising the cycle (first-order representation)



- Graph of T_P is a fractal.
- Approximation up to arbitrary precision possible.

- Requires quite some calculation to get correct parameters in higher dimensions ...

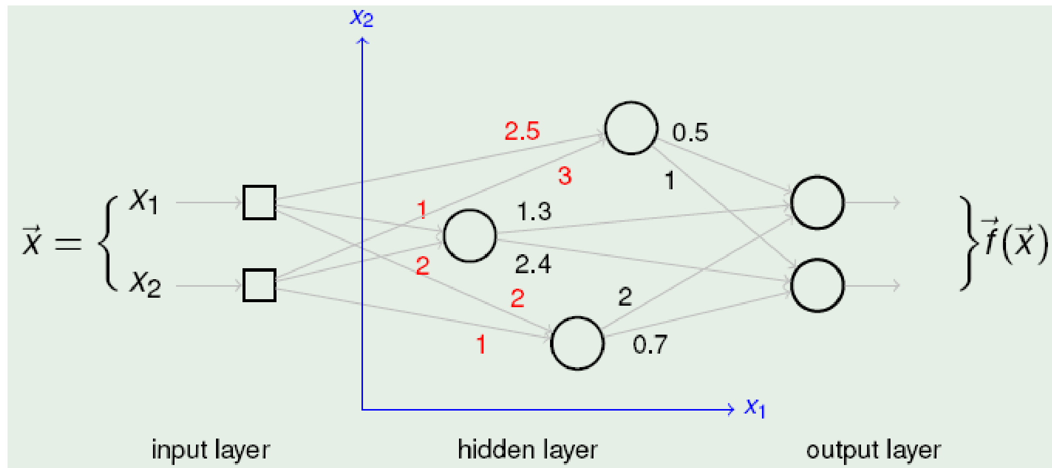


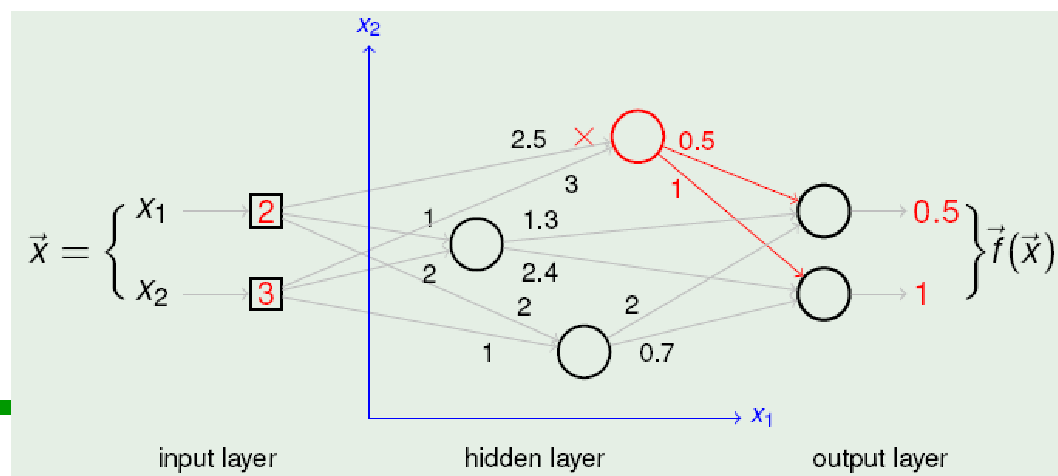


Local representation

and

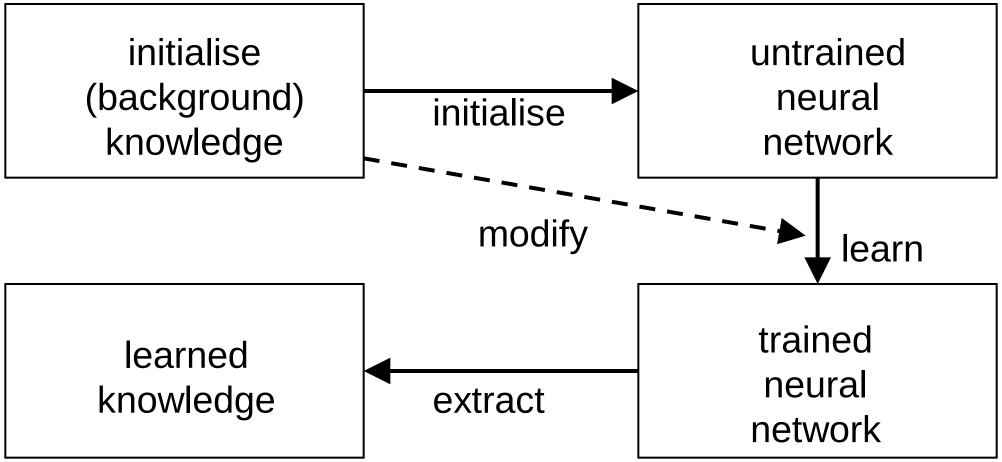
domination of output by one unit





Realising the cycle: learning

- Reuse of standard network architecture allows to use known and powerful learning methods.
  - Backpropagation
  - We merged in techniques from Supervised Growing Neural Gas (SGNG) [Fritzke 1998].



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Realising the cycle: Implementation

- Bader & Witzel, first prototype
- JDK 1.5 unter Eclipse.
- Merging of techniques above and SGNG.
  Fine Blend system.
- Radial basis function network approximating T_P.
- Very robust with respect to noise and damage.
- Trainable using a version of backpropagation together with techniques from SGNG (Supervised Growing Neural Gas).

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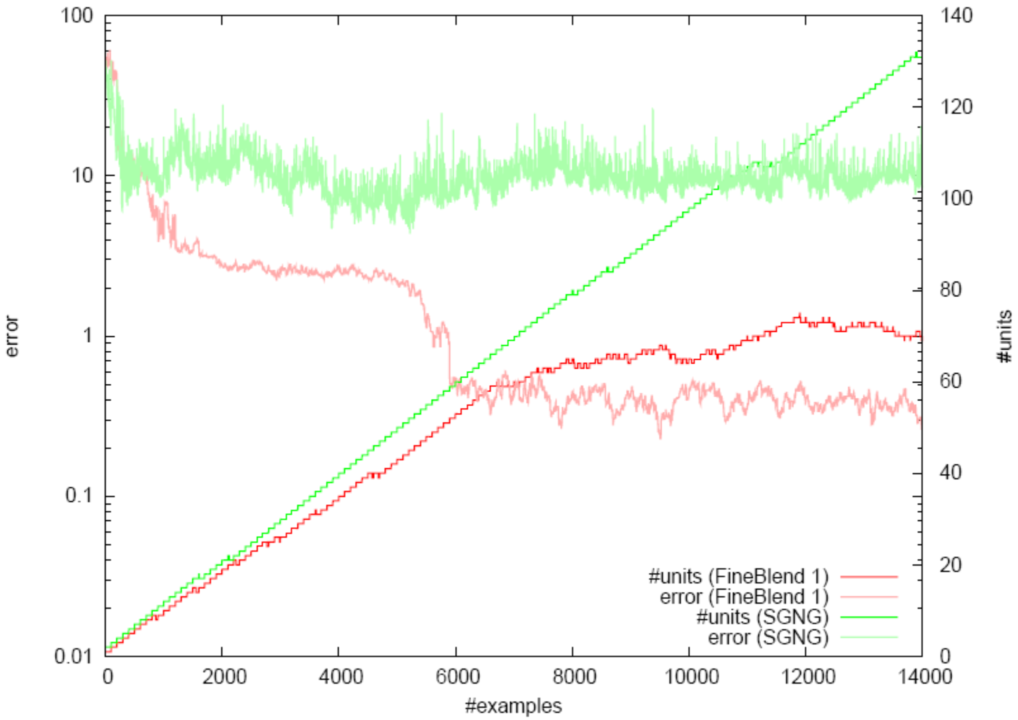
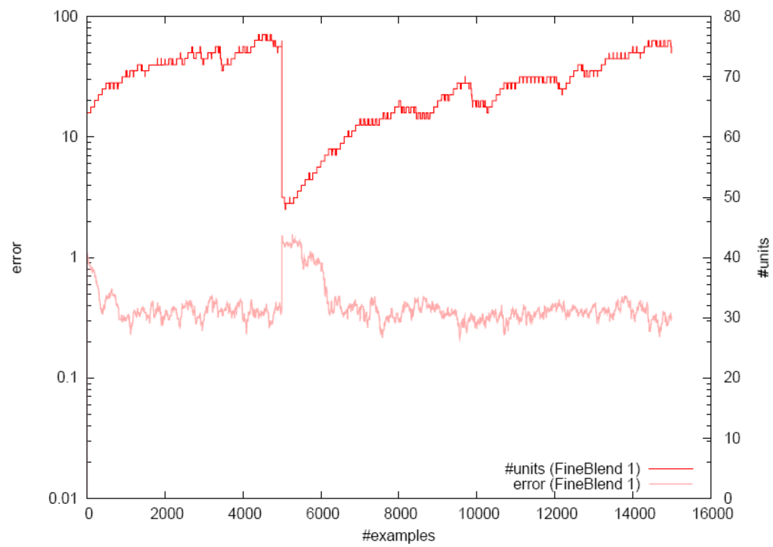
Fine blend vs. SGNG



unit failure

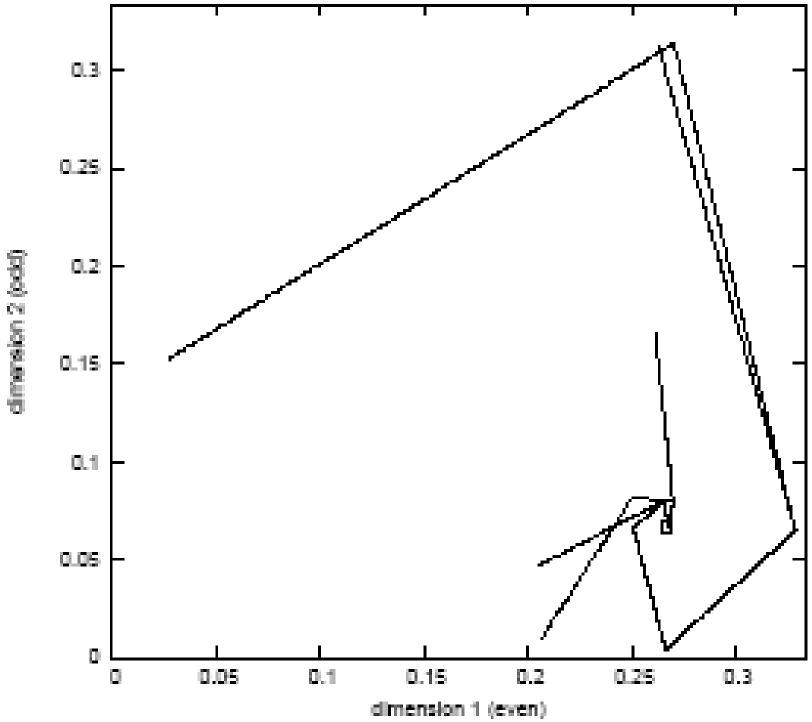


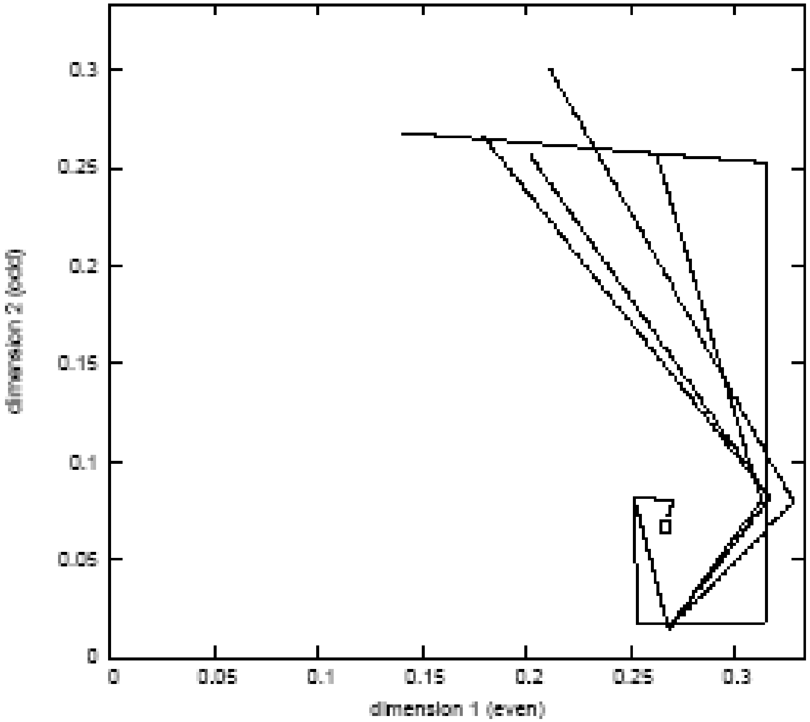
target: e(0).
e(s(X)) Ã o(X).
o(X) Ã :e(X)

initial: e(s(X)) Ã :o(X)
e(X) Ã e(X)

Iterating Random Inputs

We observe convergence to unique supported model of the program.





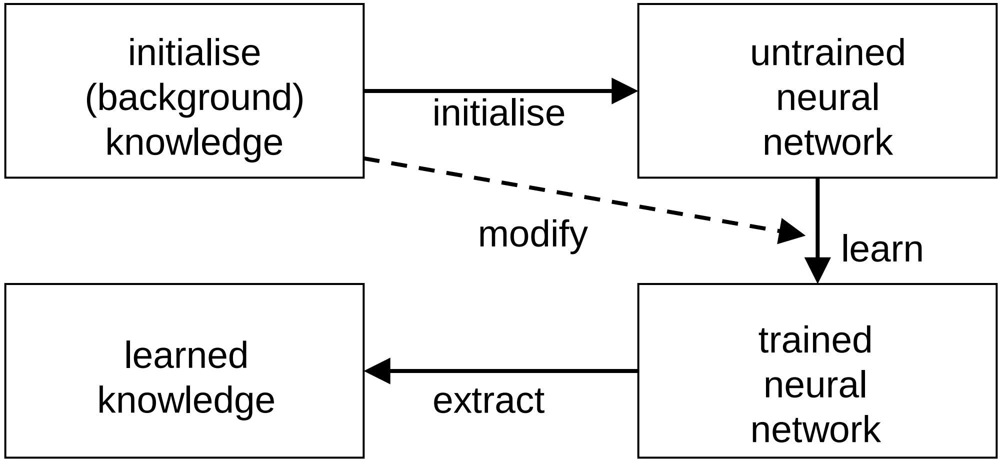
Realised integration

- Neural
  - trainable by backpropagation
  - robust
- Symbolic
  - computes logical model

☺

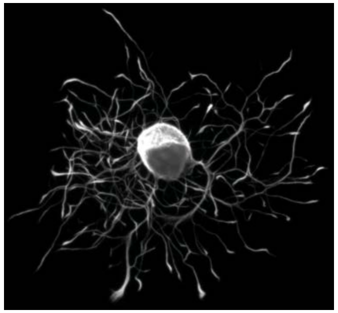
Realising the cycle: Extraction of symbolic knowledge

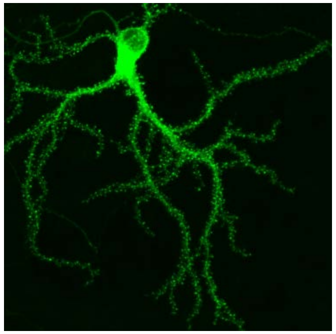
- Extraction of PL-knowledge from trained neural networks has never been attempted before.
- Idea: Represent programs and nets in $\mathbb{R}^n$ (with n = number of weights in net) and search for best approximators using suitable metrics on vectors.



Contents

1. Why neural-symbolic integration?
2. Earlier work
3. The neural-symbolic learning cycle
4. Propositional fixation
5. The cycle for first-order logic
    a. The Core Method
    b. Realising the cycle
6. **Outlook**





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Outlook

Short term:

- Further experiments and evaluations.
- Develop and realise extraction method.
- Develop concrete application scenarios.
- Realise learning under background knowledge.

Medium and long term:

- Carry over to other KRR paradigms, e.g. DLs.
- Develop integrated connectionist learning and reasoning for cognitive systems applications.

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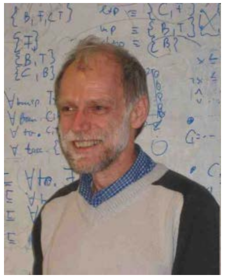
Related work I

- There is hardly any work on first-order neural-symbolic integration.
- M. Lane, A. Seda. Some Aspects of the Integration of Connectionist and Logic-Based Systems. Information, 9(4)(2006), 551-562.
  - Based on the propositional Core Method: Approximation of first-order programs by a finite number of ground instantiated clauses.
  - Purely theoretical.

Related work II



- H. Gust, K.-U. Kühnberger, P. Geibel. Learning Models of Predicate Logical Theories with Neural Networks Based on Topos Theory. In P. Hitzler, B. Hammer (eds.). Perspectives of Neural-Symbolic Integration, Studies in Computational Intelligence 77, Springer, 2007, pp. 233-264.
  – variable-free representation using category theory
  – learns corresponding models
  – running system

Related work III


- Using Bilattice-based annotated logic programs

- Propositional + first-order. Basically a lifting of the Hölldobler & Kalinke approach.

- No running system available

- [Komendantskaya, Seda, 2006]

Related work IV

- Connectionist realisation of proof-theory
- Specifically, SLD-resolution
- Tough ...
- [Komendantskaya 2007, 2008] and ongoing

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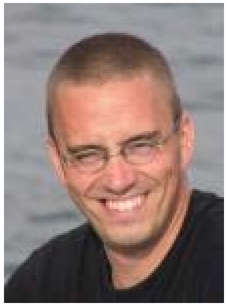
Critical Questions

- The brain doesn't use logic.
  - Well – yes. Logic is a (coarse) model. Like Newtonian physics is a coarse model.
  - We DO NEED more neuroscience input!
- The "infinity" discussion doesn't apply to the brain.
  - Well – yes. But give me something better.
- So where do you want to apply all this?
  - Good question. We currently have a hammer. We need to find some suitable nails.
  - But we DO HAVE one of the first two approaches to first-order neural-symbolic integration after 10 years of searching for it!!!!

Thank you for your attention


Collaborators

please visit
http://www.neural-symbolic.org

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- Anthony K. Seda
- Andreas Witzel




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- S. Bader, A.S. d'Avila Garcez and P. Hitzler, Computing First-Order Logic Programs by Fibring Artificial Neural Networks. In: I. Russell, Z. Markov (Eds.): Proceedings of FLAIRS05, Clearwater Beach, Florida, USA. AAAI Press 2005, May 2005, pp. 314-319.

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- S. Bader, P. Hitzler and A. Witzel, Integrating First Order Logic Programs and Connectionist Systems - A Constructive Approach, In: Proceedings of the IJCAI-05 Workshop on Neural-Symbolic Learning and Reasoning, NeSy'05, Edinburgh, UK, August 2005.
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