Neural-Symbolic Integration - Fragments

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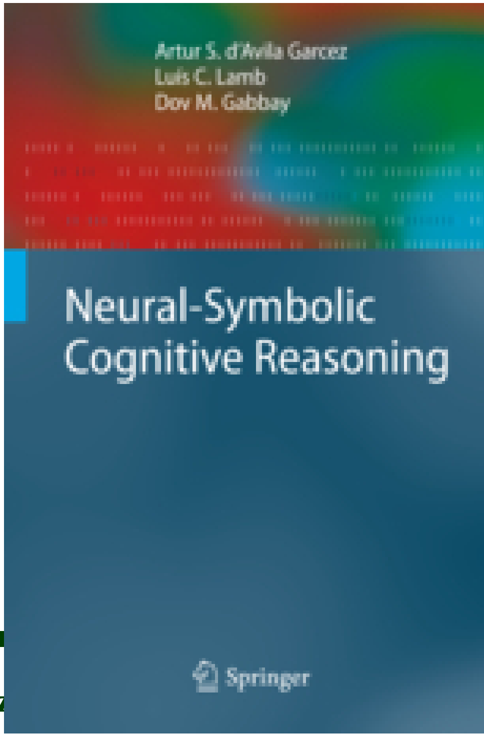
Activities

- http://neural-symbolic.org/ (hosted by me)
- Annual Workshop series "Neural-Symbolic Learning and Reasoning" at major conferences (IJCAI, AAAI, ECAI) since 2005. – initiated by me (approaching Artur Garcez)

- Corresponding Association (steering committee) established 2014.
- Dagstuhl Seminars since 2008, CoCo workshop at NIPS since 2015, tutorials, summer schools, etc.
- Books (Hammer & Hitzler, eds., 2007; Garcez et al. 2009)



Why neural-symbolic?

- **Cognitive Science: Understanding (and modeling?) human cognitive abilities**

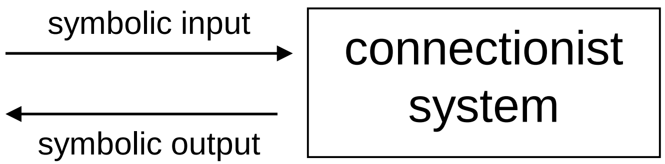
- **Computer Science: Improving and understanding connectionist machine learning systems.**

Neural: Refers to artificial neural networks (aka connectionist systems), which are sub-symbolic.

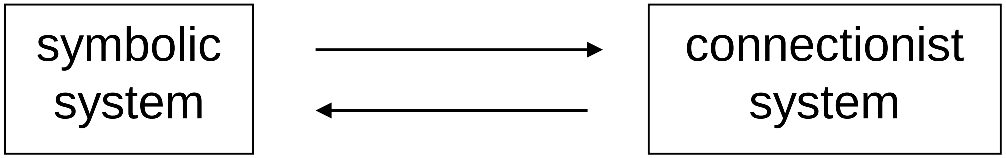
Symbolic: Refers (generally) to structured data (including trees, graphs), and (more narrowly, and for me) to logical knowledge representation.

Hybrid vs. Integrated Approach

integrated



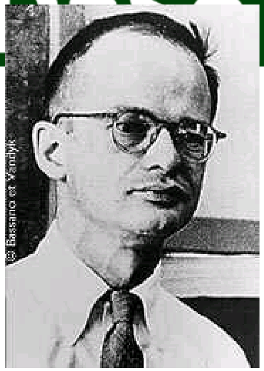
hybrid



The four main problems of NeSy

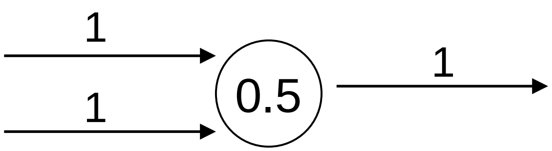
- Connectionist representation of symbolic knowledge.
- Extraction of symbolic knowledge from trained artificial neural networks.
- Connectionist learning of symbolic knowledge.
- Connectionist learning under background knowledge.

Earlier work



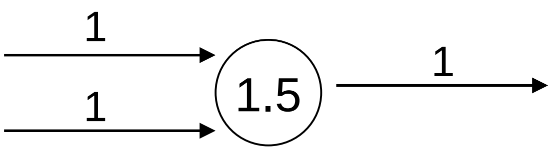
- McCulloch & Pitts 1943
  - Neurons with binary activation functions.
  - Modelling of propositional connectives.
  - Networks equivalent to finite automata.

Values 0 („false“) and 1 („true“) being propagated.



disjunction

Simultaneous update of all nodes in network.



conjunction

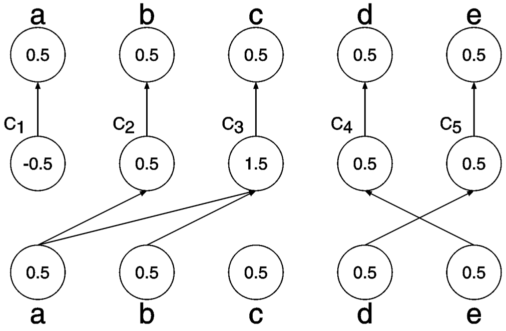
negation

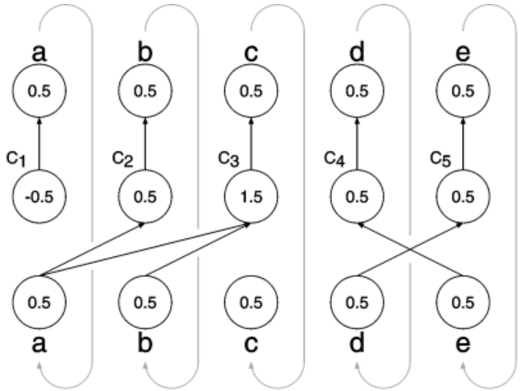
The propositional *Core Method*

- Hölldobler & Kalinke 1994
  - Extends the approach by McCulloch & Pitts.
  - Representation of propositional logic programs and their semantics.
  - „Massively parallel reasoning.“

logic program → core net → recurrent net

$a \leftarrow$
$b \leftarrow a$
$c \leftarrow a \wedge b$
$d \leftarrow e$
$e \leftarrow d$



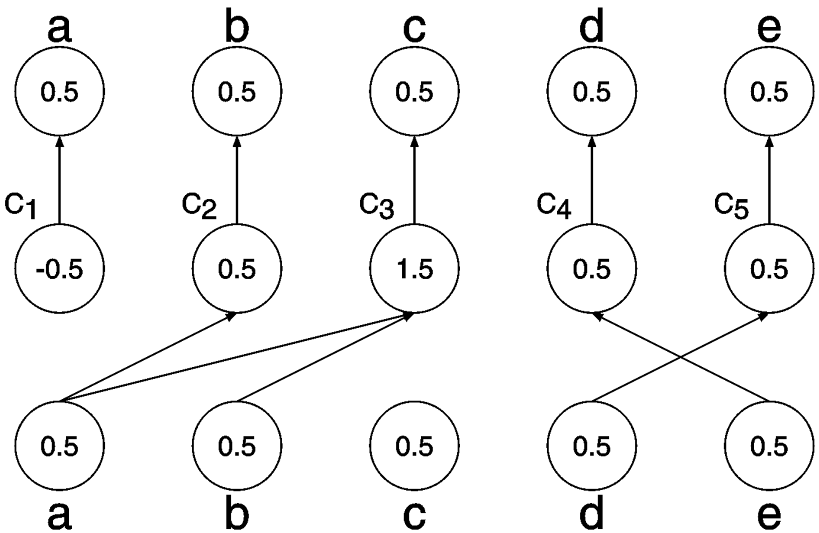


The propositional *Core Method*

Logic program P ⟶ core net

$a \leftarrow$
$b \leftarrow a$
$c \leftarrow a \wedge b$
$d \leftarrow e$
$e \leftarrow d$

⟶

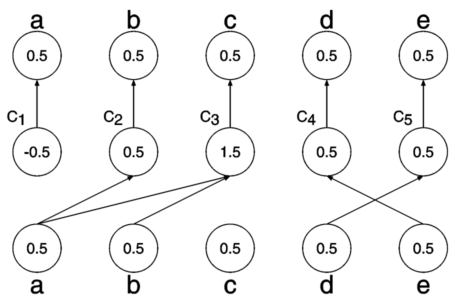


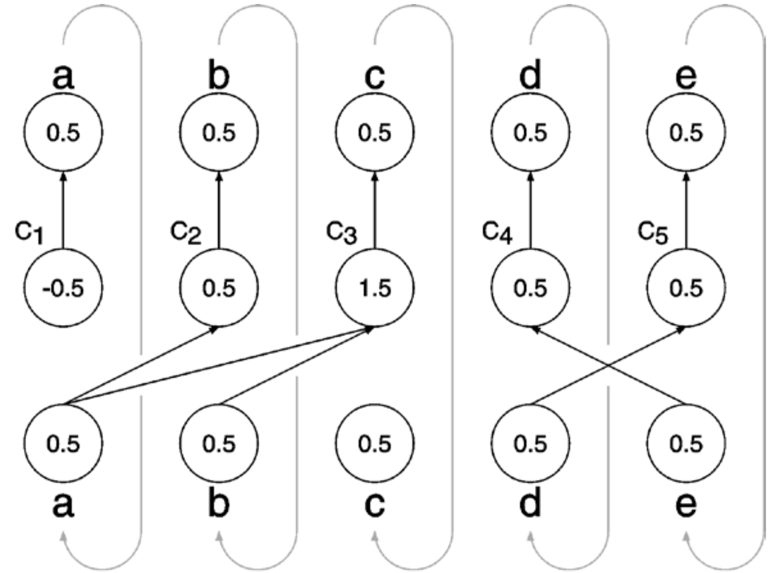
- Update „along implication“.
- Corresponds to computing the semantic operator T_P.
- T_P represents meaning (semantics) of P through its fixed points.

The propositional *Core Method*

core net

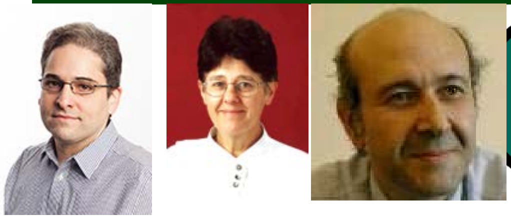
recurrent net



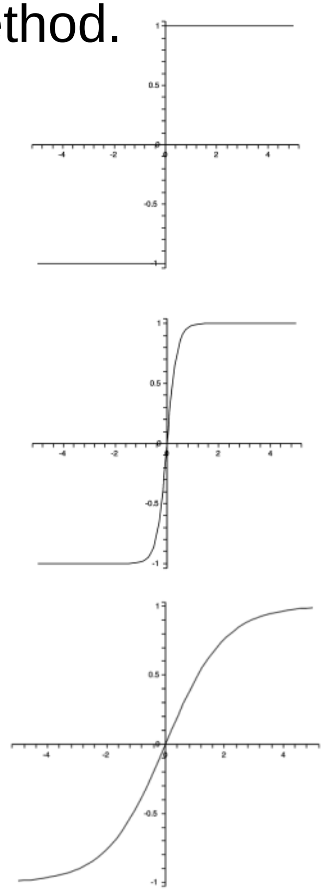


- Repeated updates along layers corresponds to iterations of the semantic operator.
- Semantics of the program (= fixed point of the operator) can be computed in a parallel manner.

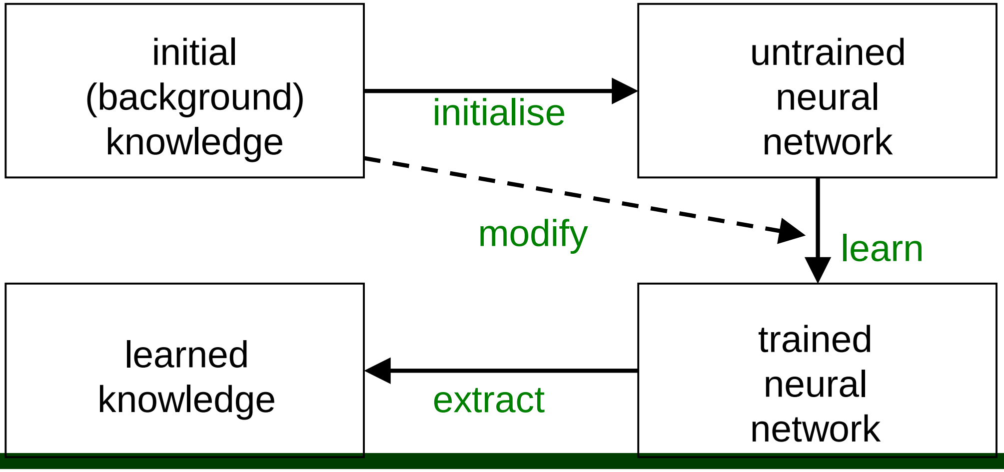
CILP: Conn. Ind. Logic Prog.



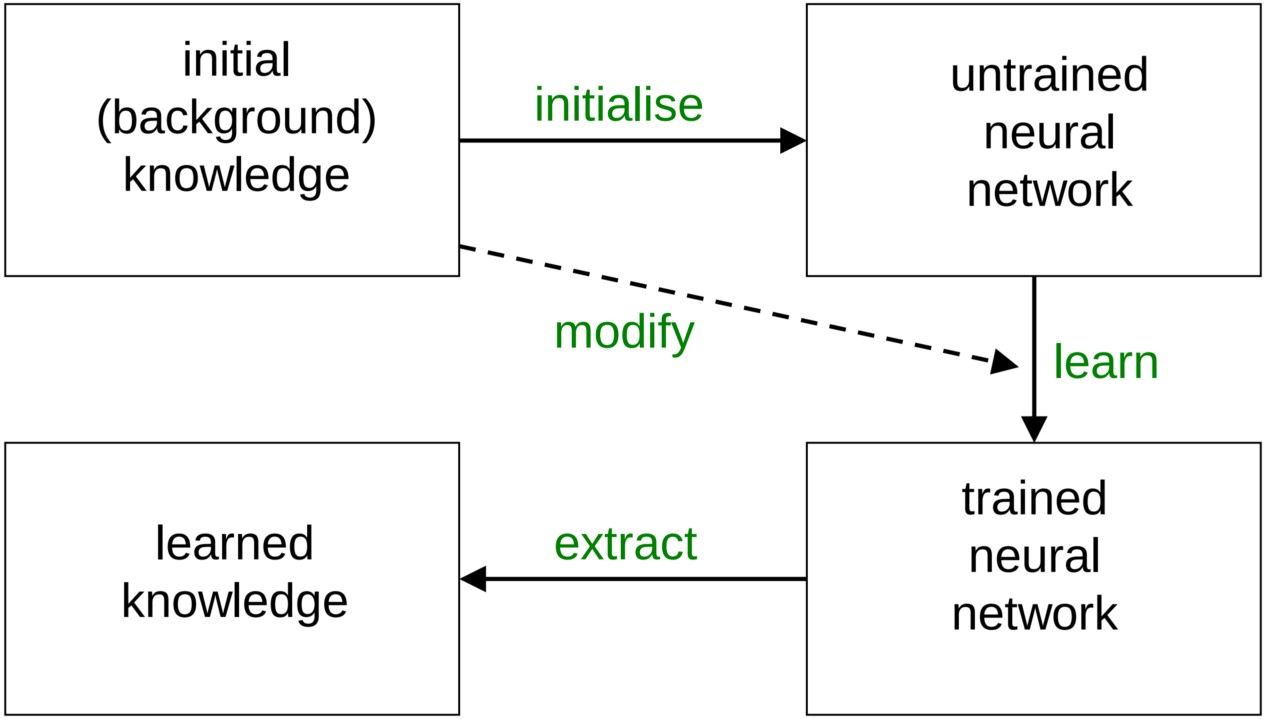
- Garcez & Zaverucha 1999
  Garcez, Broda & Gabbay 2001
- Development of a learning paradigm from the Core Method.
- Required: differentiable activation function.
  - Allows learning with standard methods.
  - Backpropagation algorithm.



- Establishing the *neural-symbolic learning cycle*.



The neural-symbolic learning cycle



The four main problems of Neural-symbolic Integration.

Extraction

In this case: extracting propositional rules.

General idea:

- **Input value 1 interpreted as “true”, value 0 as “false”**
- **Outputs interpreted as true or false according to a threshold**
- **I.e. network function maps binary vectors.**

Garcez et al, 2001: By weight analysis (layer by layer) under differentiable activation functions. Possible in principle but intricate and, arguably, the resulting rule sets are usually rather difficult to understand.



Lehmann, Bader, Hitzler, 2010: Black-box approach (looking at inputs and outputs only).



Lehmann, Bader, Hitzler, 2010



For every function

$$f : \{0,1\}^n \rightarrow \{0,1\}^k$$

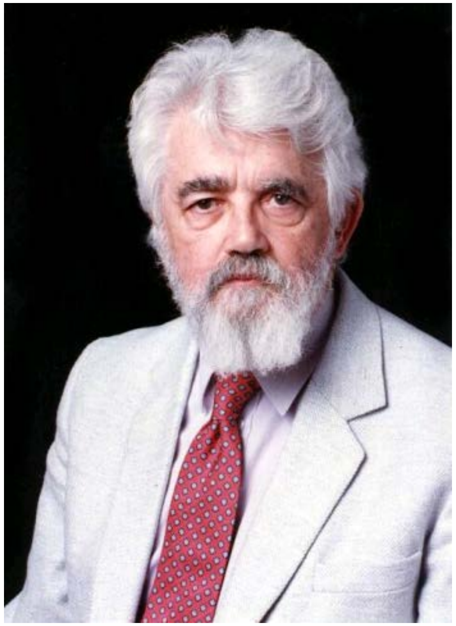
there is a unique reduced set of positive propositional rules which capture exactly the function f.

Reduced means: no redundancies, and as small as possible.

Conectionism and first-order predicate logic

- Connectionist representation of PL-knowledge very hard to realise.
  McCarthy 1988: „Propositional fixation.“

  We need to capture the infinite in a finite way.



  – infinite ground instantiations

  $$\mathrm{male}(x) \wedge \mathrm{hasSon}(x, \mathrm{son}(x)) \rightarrow \mathrm{father}(x)$$

  – term representations
    member(X, [a,b,c | [d,e]])
  – variable bindings

  $$\mathrm{male}(x) \wedge \mathrm{hasSon}(x, y) \rightarrow \mathrm{father}(x)$$

PL Core Method

- Hölldobler, Kalinke, Störr 1999
  Hitzler, Hölldobler, Seda 2004

- Idea:
  - Use results by Funahashi 1989: „Every continuous function on the reals is approximable by standard feedforward networks. “
  - Hence: Consider logic programs for which T_P-operator is continuous in this sense.

Cantor Space

$T_P: I_P \to I_P$ is locally finite

iff

T_P is continuous in Cantor space.

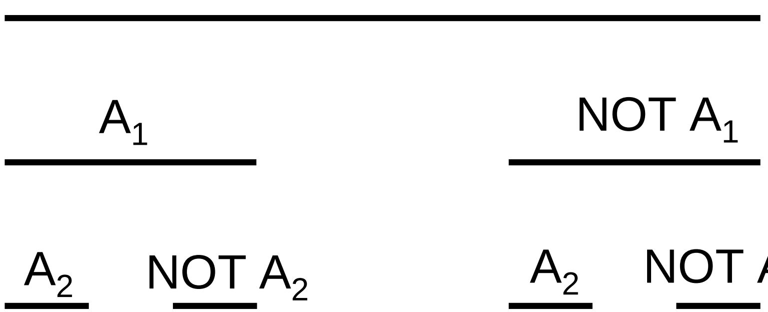
- **Cantor-continuity is continuity wrt. the Cantor topology on the Cantor set.**
- **The Cantor topology is homeomorphic to the prefix-distance on (infinite) binary trees.**
- **The Cantor topology is homeomorphic to the subspace topology which is induced on a subset of IR which is compact, totally disconnected and dense in itself.**

Cantor Space

- There are (uncontably) many homeomorphisms which map I_P with the Cantor topology into suitable subsets of R.
- Locally finiteness is a logical (topology-free) characterisation of logic programs which can be represented in a a connectionist way in the sense of Funahashi.
- Problem: this argumentation is not constructive!

A_1, A_2, ... enumeration of Herbrand base

Elements of Cantor Set identifiable with interpretations



Cantor Space

Interpretations with Cantor topology

Homeomorphism

Cantor space as compact subspace of IR

$$\begin{tikzcd} (I_P,Q) \arrow[r, "T_P"] \arrow[d, "i"] & (I_P,Q) \arrow[d, "i"] \\ \text{Cantor} \arrow[r, "i(T_P)"] & \text{Cantor} \end{tikzcd}$$

Georg Cantor

Cantor topology as bridge

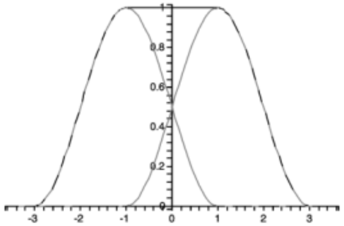
- **Connectionist side:**
  - **Cantor topology is a subtopology of the usual topology on the real numbers**

- **Logic Programming side:**
  - **Cantor topology captures useful notions of convergence of semantic operators, e.g.**
    If T_P^n -> I (for $n \to 1$), then I is a model of P.

Representation

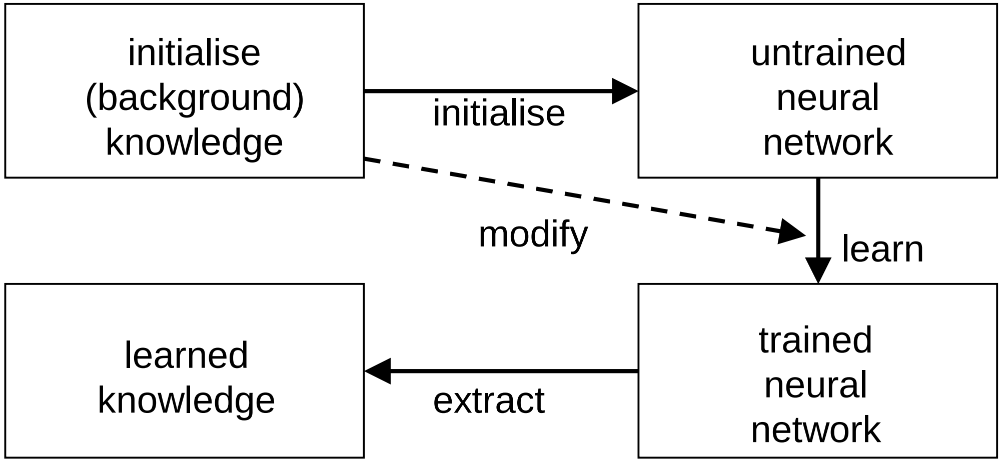
- Bader, Hitzler, Hölldobler, Witzel – IJCAI-07
  - Algorithm for the approximate construction of neural networks from logic programs.
  - Realised for
    - RBS nets with triangular activation function
    - RBF nets with raised cosine activation function

$$\tau_{w,h,m}(x) = \begin{cases} \frac{h}{2} \cdot \left(1 + \cos\left(\frac{\pi(x-m)}{w}\right)\right) & \text{if } |x-m| < w \\ 0 & \text{otherwise} \end{cases}$$



Realising the cycle: learning

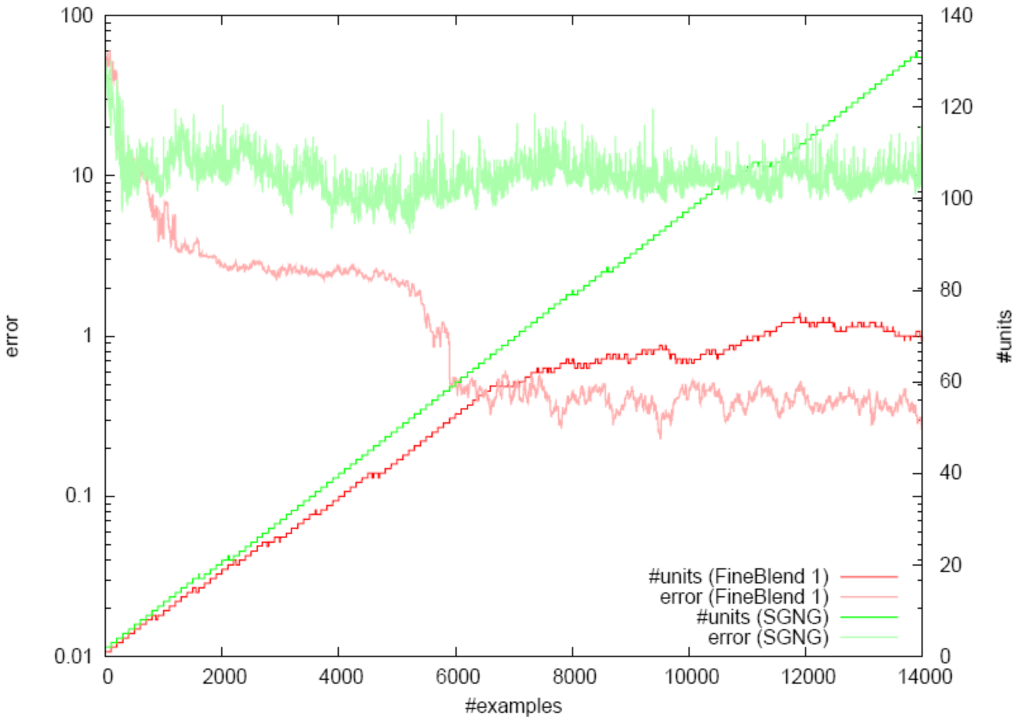
- **Reuse of standard network architecture allows to use known and powerful learning methods.**
  - **Backpropagation**
  - **We merged in techniques from Supervised Growing Neural Gas (SGNG) [Fritzke 1998].**



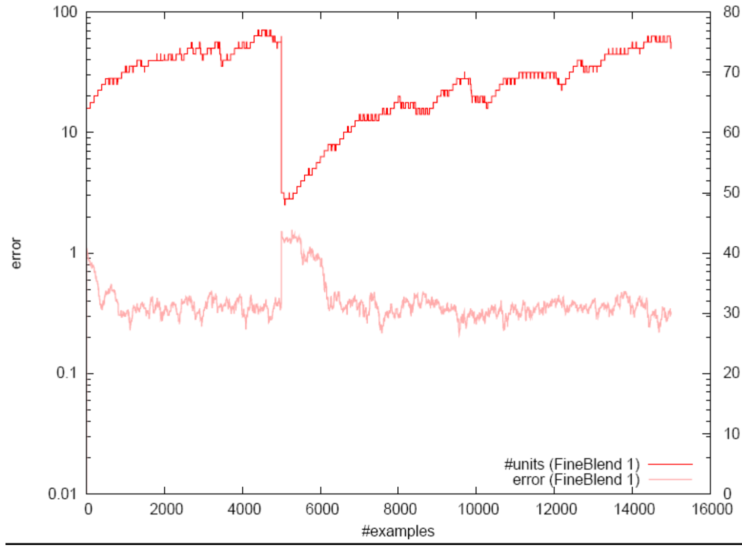
Realising the cycle: Implementation

- **Bader & Witzel, first prototype**
- **JDK 1.5 unter Eclipse.**
- **Merging of techniques above and SGNG.**
  ***Fine Blend* system.**
- **Radial basis function network approximating T_P.**
- **Very robust with respect to noise and damage.**
- **Trainable using a version of backpropagation together with techniques from SGNG (Supervised Growing Neural Gas).**

Fine blend vs. SGNG



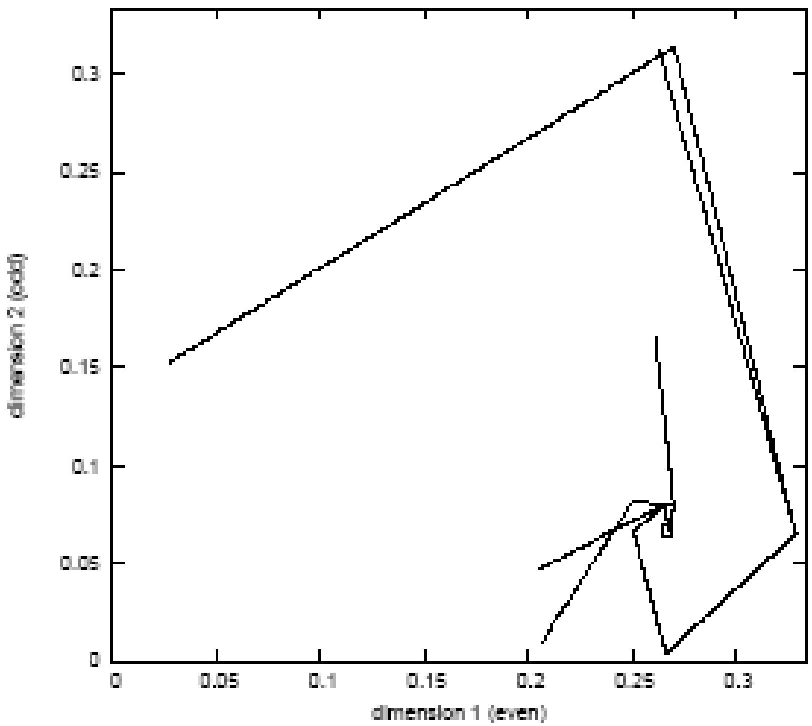
unit failure

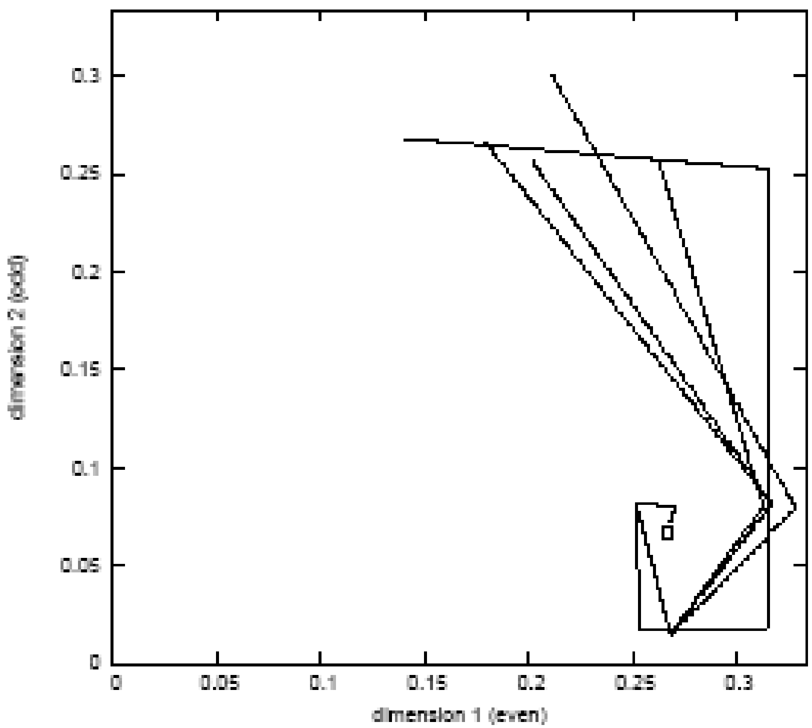


target:	e(0).	
	e(s(X))	<- o(X).
	o(X)	<- NOT e(X)
initial:	e(s(X))	<- NOT o(X)
	e(X)	<- e(X)

Iterating Random Inputs

We observe convergence to unique supported model of the program.





Realised integration

- **Neural**
  - **trainable by backpropagation**
  - **robust**
- **Symbolic**
  - **computes logical model**

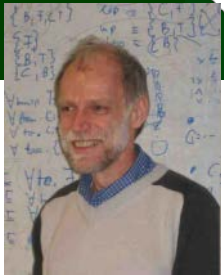
☺

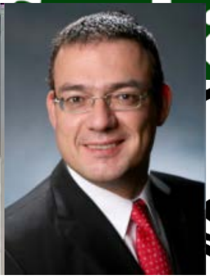
But: Very small toy problems only!

Related work I

- **There is hardly any work on first-order neural-symbolic integration.**
- **M. Lane, A. Seda. Some Aspects of the Integration of Connectionist and Logic-Based Systems. Information, 9(4)(2006), 551-562.**
  - **Based on the propositional Core Method: Approximation of first-order programs by a finite number of ground instantiated clauses.**
  - **Purely theoretical.**

Related work II



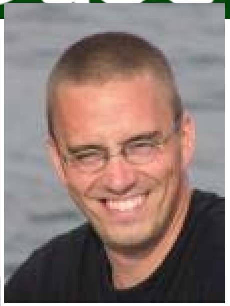


- **H. Gust, K.-U. Kühnberger, P. Geibel. Learning Models of Predicate Logical Theories with Neural Networks Based on Topos Theory. In P. Hitzler, B. Hammer (eds.). Perspectives of Neural-Symbolic Integration, Studies in Computational Intelligence 77, Springer, 2007, pp. 233-264.**
  - **variable-free representation using category theory**
  - **learns corresponding models**
  - **running system**

Collaborators

DaSe Lab

Thanks!



- Sebastian Bader
- Artur S. d’Avila Garcez
- Barbara Hammer
- Steffen Hölldobler
- Kai-Uwe Kühnberger
- Jens Lehmann
- Anthony K. Seda
- Andreas Witzel

please visit
http://www.neural-symbolic.org

WRIGHT STATE UNIVERSITY

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- P. Hitzler, S. Bader and A. S. d'Avila Garcez, Ontology leaning as a use case for neural-symbolic integration, In: Proceedings of the IJCAI-05 Workshop on Neural-Symbolic Learning and Reasoning, NeSy'05, Edinburgh, UK, August 2005.
- J. Lehmann, S. Bader and P. Hitzler, Extracting reduced logic programs from artificial neural networks, In: Proceedings of the IJCAI-05 Workshop on Neural-Symbolic Learning and Reasoning, NeSy'05, Edinburgh, UK, August 2005.
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- S. Bader, P. Hitzler, S. Hölldobler. Connectionist Model Generation: A First-Order Approach. Neurocomputing 71, 2008, 2420-2432.
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- Pascal Hitzler, Kai-Uwe Kühnberger, Facets of Artificial General Intelligence. Künstliche Intelligenz 2/09, 58-59, 2009.
- Pascal Hitzler, Kai-Uwe Kühnberger, The Importance of Being Neural-Symbolic - A Wilde Position. In: Ben Goertzel, Pascal Hitzler, Marcus Hutter (eds.), Artificial General Intelligence. Second Conference on Artificial General Intelligence, AGI 2009, Arlington, Virginia, USA, March 6-9, 2009. Proceedings, pp. 208-209.

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- Artur d'Avila Garcez, Tarek R. Besold, Luc de Raedt, Peter Földiak, Pascal Hitzler, Thomas Icard, Kai-Uwe Kühnberger, Luis C. Lamb, Risto Miikkulainen, Daniel L. Silver, Neural-Symbolic Learning and Reasoning: Contributions and Challenges. In: Andrew McCallum, Evgeniy Gabrilovich, Ramanathan Guha, Kevin Murphy (eds.), Proceedings of the AAAI 2015 Spring Symposium on Knowledge Representation and Reasoning: Integrating Symbolic and Neural Approaches. Technical Rport SS-15-03, AAAI Press, Palo Alto, 2015.