

# Neuro-Symbolic Artificial Intelligence: A Brief History, and Recent Advances



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<http://www.daselab.org>



# Neuro-Symbolic

# Some Background

**Workshop Series on Neural-Symbolic Learning and Reasoning, since 2005.  
Joint with Artur d'Avila Garcez.**

<http://neural-symbolic.org/>

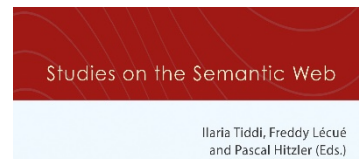
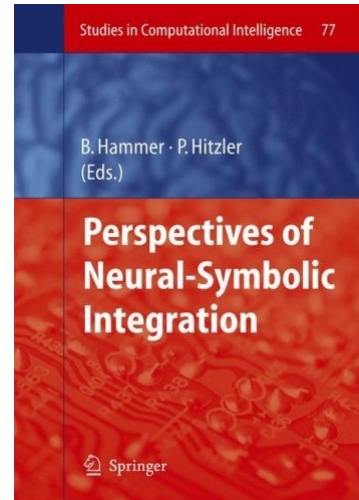
**Barbara Hammer and Pascal Hitzler (eds), Perspectives of  
Neural-Symbolic Integration, Springer, 2007**

**Neural-Symbolic Learning and Reasoning: A Survey and Interpretation**

**Tarek R. Besold, Artur d'Avila Garcez, Sebastian Bader,  
Howard Bowman, Pedro Domingos, Pascal Hitzler,  
Kai-Uwe Kuehnberger, Luis C. Lamb, Daniel Lowd,  
Priscila Machado Vieira Lima, Leo de Penning, Gadi Pinkas,  
Hoifung Poon, Gerson Zaverucha**

<https://arxiv.org/abs/1711.03902> (2017)

**Ilaria Tiddi, Freddy Lecue, Pascal Hitzler (eds.), Knowledge Graphs  
for eXplainable Artificial Intelligence: Foundations, Applications and  
Challenges. Studies on the Semantic Web Vol. 47, IOS Press, 2020.**



# 2022 Book

## Neuro-symbolic Artificial Intelligence: The State of the Art

Pascal Hitzler and Md Kamruzzaman Sarker, editors

Frontiers in AI and Applications Vol. 342, IOS Press, Amsterdam, 2022

<https://www.iospress.com/catalog/books/neuro-symbolic-artificial-intelligence-the-state-of-the-art>

Frontiers  
in  
Artificial  
Intelligence  
and  
Applications

NEURO-SYMBOLIC  
ARTIFICIAL  
INTELLIGENCE:  
THE STATE  
OF THE ART

Edited by  
Pascal Hitzler  
Md Kamruzzaman Sarker

IOS Press

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# Neuro-symbolic AI



## Publications on neuro-symbolic AI in major conferences (research papers only):

conference	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	total
ICML	0	0	0	0	0	1	3	2	5	6	17
NeurIPS	0	0	0	0	0	0	0	4	2	4	10
AAAI	0	0	0	0	0	1	0	1	1	1	4
IJCAI	1	0	0	0	0	0	2	2	0	2	7
ICLR	N/A	N/A	0	0	0	0	1	1	1	3	6
total	1	0	0	0	0	2	6	10	9	16	44

See

**Md Kamruzzaman Sarker, Lu Zhou, Aaron Eberhart, Pascal Hitzler**  
**Neuro-Symbolic Artificial Integration: Current Trends**  
**AI Communications 34 (3), 197-209, 2022.**

# New Book for 2023

**Compendium of Neuro-Symbolic Artificial Intelligence (tentative)**



**approx. 30 chapters and 700 pages**

**Each chapter based on 2 or more related published papers.**

**Book will provide an even more comprehensive overview of the state of the art.**

**[We can still add a few chapters – see <https://daselab.cs.ksu.edu/content/call-book-chapter-proposals-compendium-neuro-symbolic-artificial-intelligence> and send your chapter proposal very quickly.]**

# Neural



- Refers to computational abstractions of (natural) neural network systems.
- Prominently includes Artificial Neural Networks and Deep Learning as machine learning paradigms.
- More generally sometimes referred to as *connectionist systems*.
  
- Prominent applications come from the machine learning world.
- And of course, there is the current deep learning hype.

# Symbolic



- Refers to (computational) symbol manipulations of all kind.
- Graphs and trees, traversal, data structure operations.
- Knowledge representation in explicit symbolic form (data base, ontology, knowledge graph)
- Inductive and statistical inference.
- Formal logical (deductive or abductive) reasoning.
- Prominent applications all over computer science, including expert systems (and their modern versions), information systems, data management, added value of data annotation, etc.
- Semantic Web data is inherently symbolic.



## Computer Science perspective:

- **Let's try to get the best of both worlds:**
  - very powerful machine learning paradigm
  - robust to data noise
  - easy to understand and assess by humans
  - good at symbol manipulation
  - work seamlessly with background (domain) knowledge
- **How to do that?**
  - Endow connectionist systems with symbolic components?
  - Add connectionist learning to symbolic reasoners?
  - ... ?

# Neuro-symbolic AI

conference	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	total
ICML	0	0	0	0	0	1	3	2	5	6	17
NeurIPS	0	0	0	0	0	0	0	4	2	4	10
AAAI	0	0	0	0	0	1	0	1	1	1	4
IJCAI	1	0	0	0	0	0	2	2	0	2	7
ICLR	N/A	N/A	0	0	0	0	1	1	1	3	6
total	1	0	0	0	0	2	6	10	9	16	44

DaSe Lab

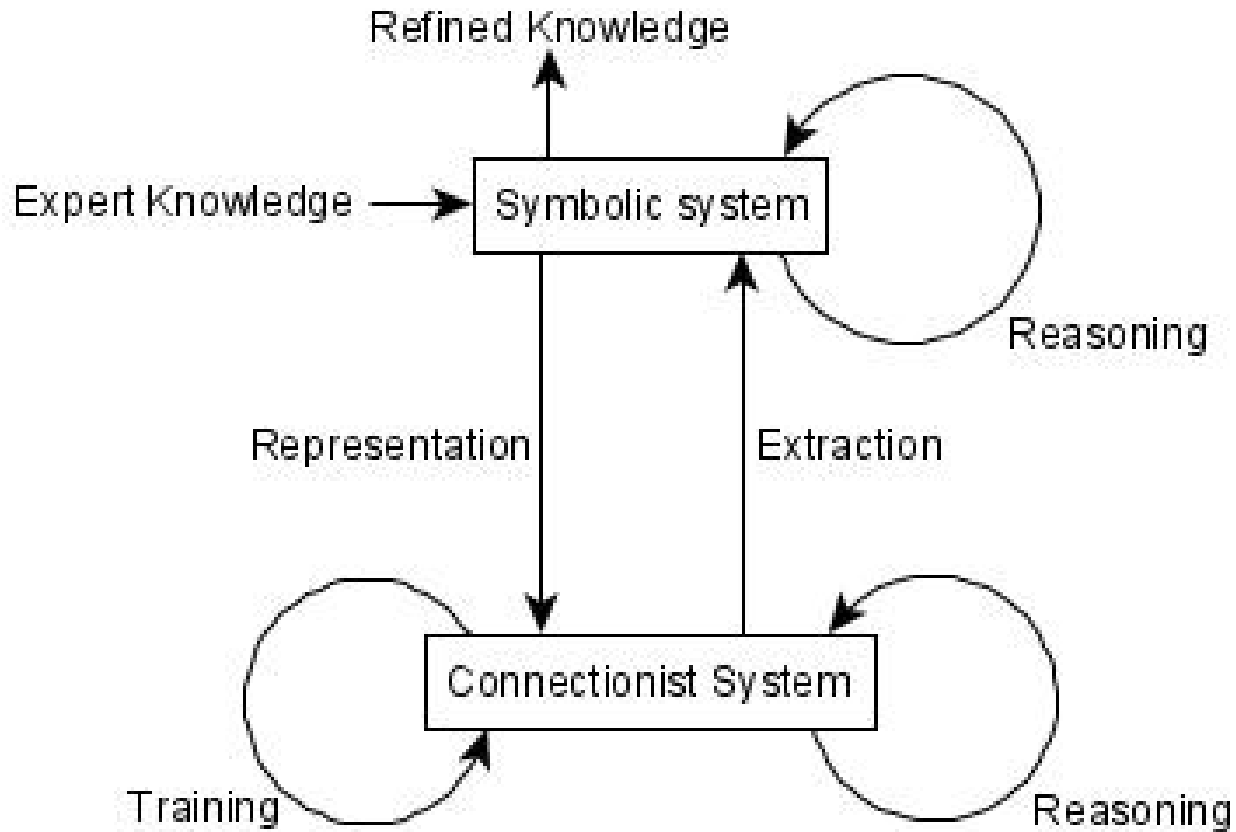
**[AI Communications 34 (3), 197-209, 2022]**

**Analysis based on**

- **structured survey from 2005 [Bader and Hitzler, Dimensions of Neural-symbolic Integration – A Structured Survey]**
- **categories presented by Henry Kautz at AAAI 2020 [cf. Kautz, AI Magazine 43, 2022, 105-125]**

**How did themes, methods, emphases change?**

# Neuro-symbolic Learning Cycle



[Bader and Hitzler 2005]



# Three “Old” Examples

# McCulloch & Pitts, 1943



- McCulloch & Pitts 1943
  - Neurons with binary activation functions.
  - Modelling of propositional connectives.
  - Networks equivalent to finite automata.

Values 0 („false“) and 1 („true“) being propagated.



disjunction

Simultaneous update of all nodes in network.



conjunction



negation

# McCulloch & Pitts follow-on

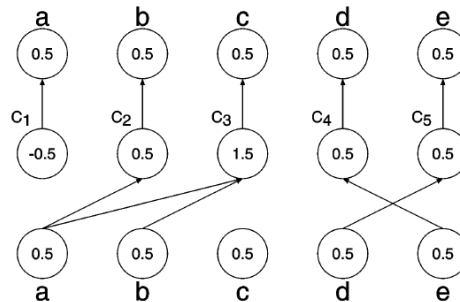
- Hölldobler & Kalinke 1994
  - Extends the approach by McCulloch & Pitts.
  - Representation of propositional logic programs and their semantics.
  - „Massively parallel reasoning.“

logic program

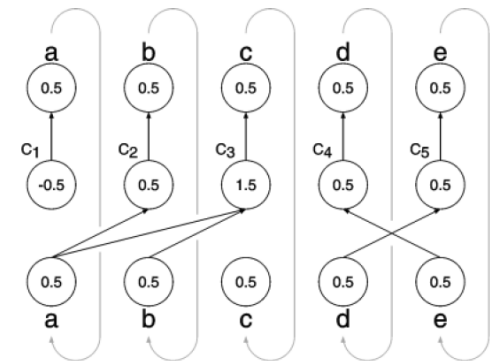
$a \leftarrow$   
 $b \leftarrow a$   
 $c \leftarrow a \wedge b$   
 $d \leftarrow e$   
 $e \leftarrow d$



core net



recurrent net



# McCulloch & Pitts follow-on

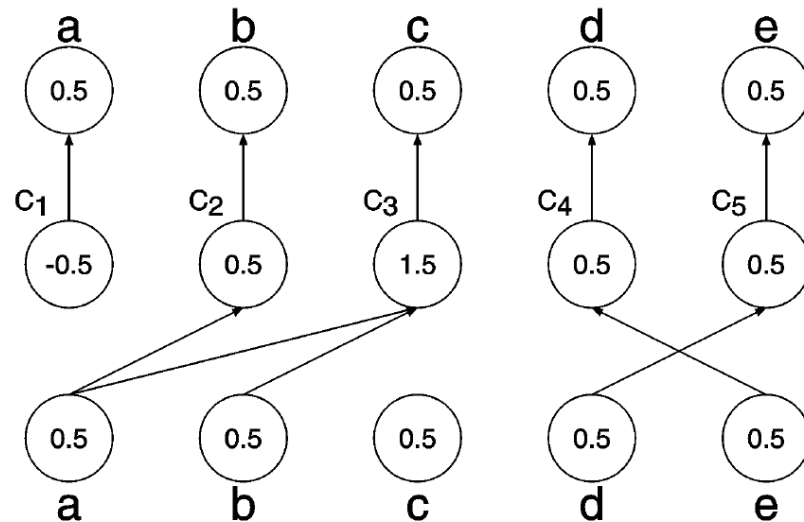


Logic program P



core net

$a \leftarrow$   
 $b \leftarrow a$   
 $c \leftarrow a \wedge b$   
 $d \leftarrow e$   
 $e \leftarrow d$



- Update „along implication“.
- Corresponds to computing the semantic operator  $T_P$ .
- $T_P$  represents meaning (semantics) of P through its fixed points.

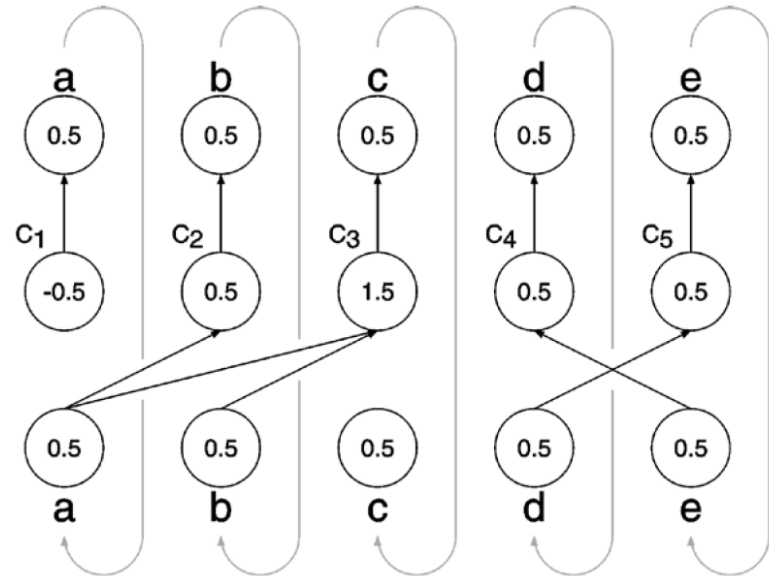
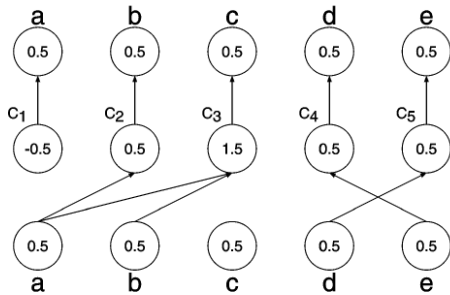
# McCulloch & Pitts follow-on



core net



recurrent net

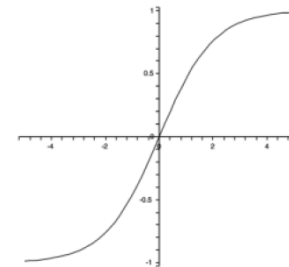
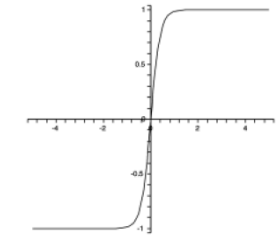
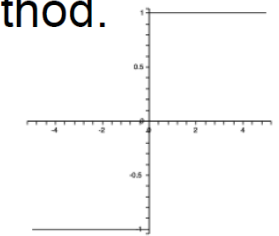


- Repeated updates along layers corresponds to iterations of the semantic operator.
- Semantics of the program (= fixed point of the operator) can be computed in a parallel manner.

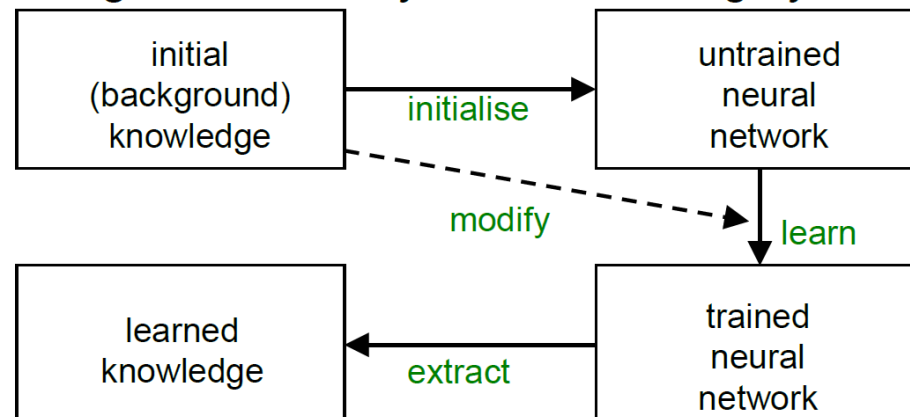


# McCulloch & Pitts follow-on

- Garcez & Zaverucha 1999  
Garcez, Broda & Gabbay 2001
- Development of a learning paradigm from the Core Method.
- Required: differentiable activation function.
  - Allows learning with standard methods.
  - Backpropagation algorithm.



- Establishing the *neural-symbolic learning cycle*.



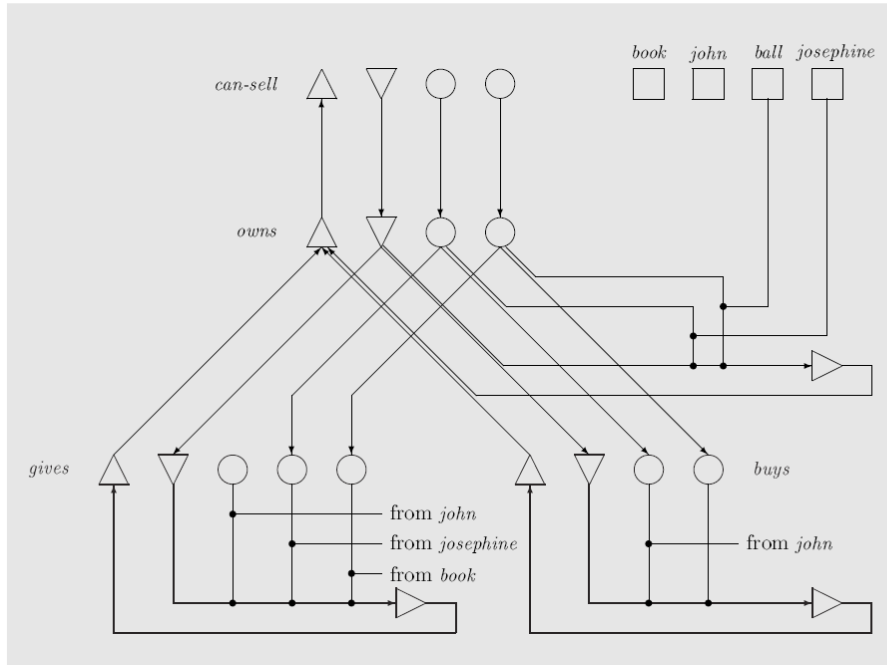
# The catch



- **This is all propositional.**
- **There's only that much you can do with propositional logic. [For what you can do, see extensive research by Artur Garcez et al.]**
- **In particular, in terms of knowledge representation and reasoning, propositional logic doesn't really get you anything useful.**

# Variable Binding

## SHRUTI



Shastri & Ajjanagadde 1993

Variable binding  
via time synchronization.

*Reflexive* (i.e. fast)  
*reasoning* possible.

Picture: Hölldobler,  
*Introduction to  
Computational Logic*, 2001

$\text{gives}(X,Y,Z) \rightarrow \text{owns}(Y,Z)$

$\text{buys}(X,Y) \rightarrow \text{owns}(X,Y)$

$\text{owns}(X,Y) \rightarrow \text{can-sell}(X,Y)$

$\text{gives}(\text{john}, \text{josephine}, \text{book})$

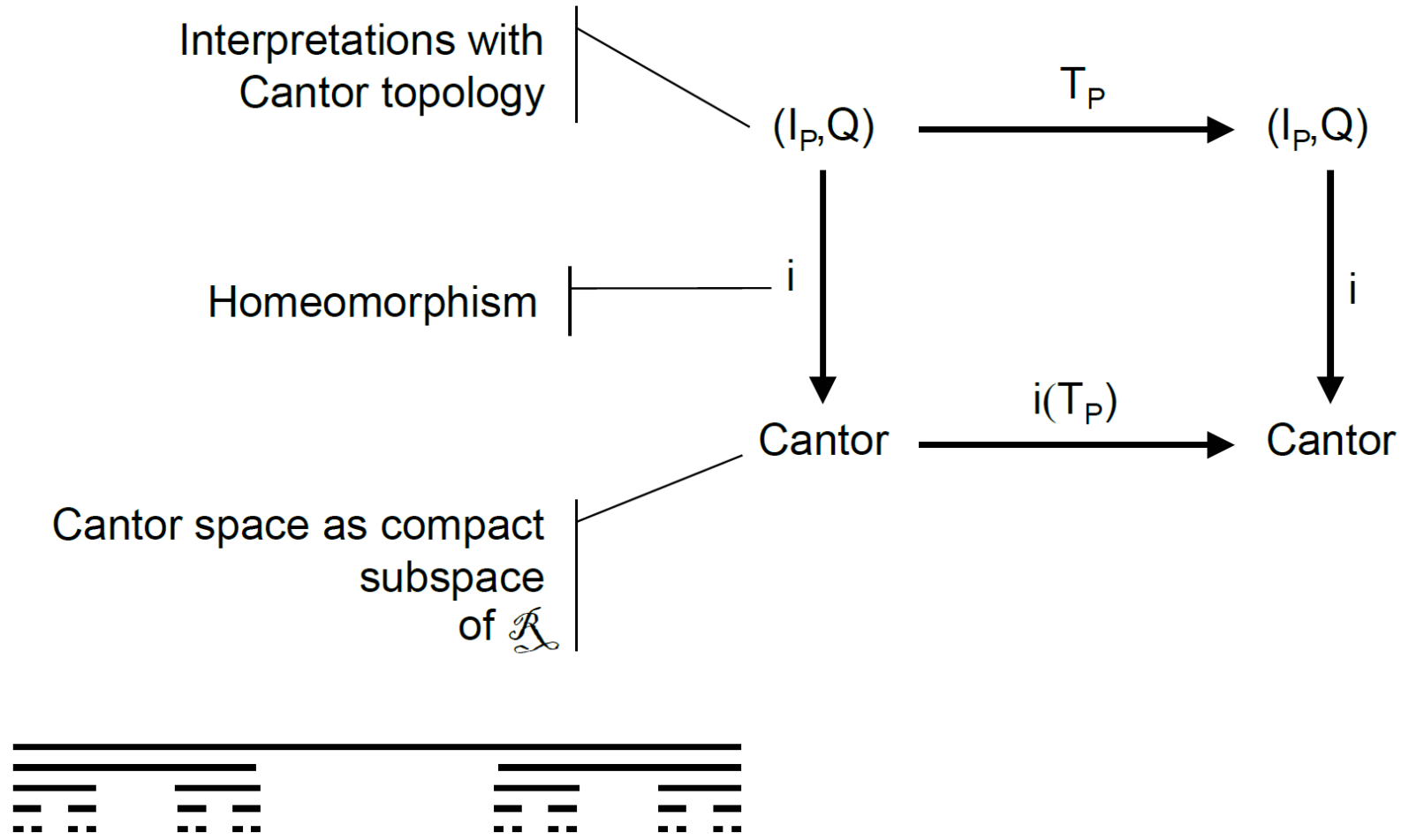
$(\exists X) \text{buys}(\text{john}, X)$

$\text{owns}(\text{josephine}, \text{ball})$

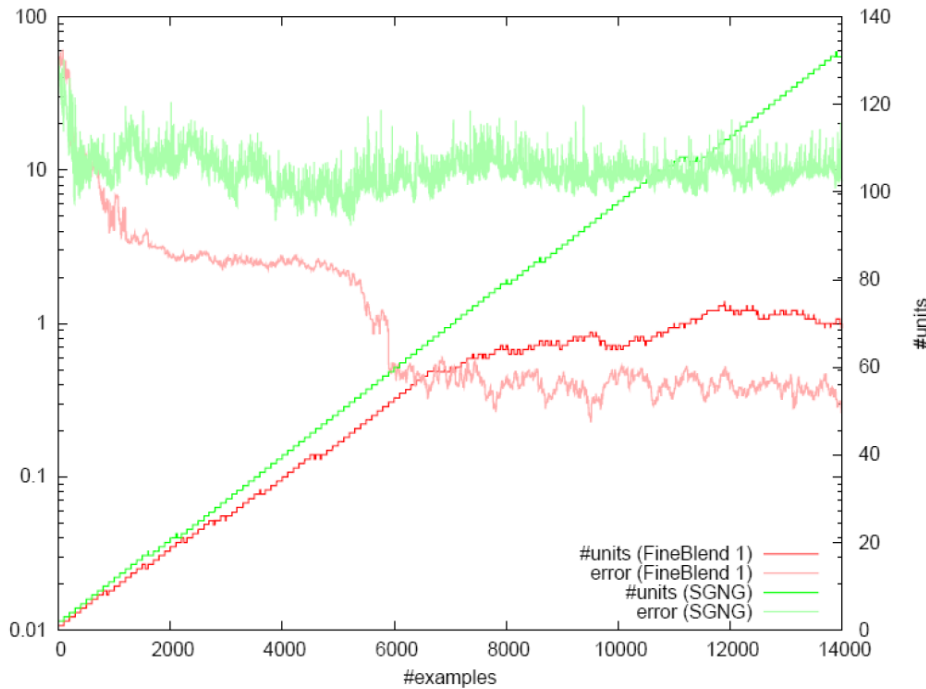
### Problems:

- It's still essentially datalog.
- It has a globally bounded reasoning depth.
- \* It doesn't really learn.

# Logic in Real Space



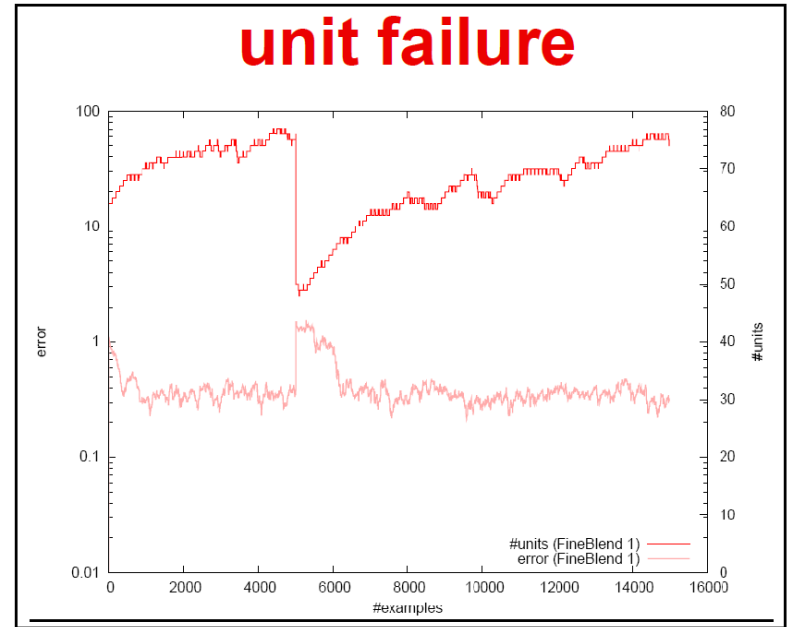
# Logic in Real Space



Architecture is mix of radial basis function network and neural gas approach.

target:  $e(0).$   
 $e(s(X)) \leftarrow o(X).$   
 $o(X) \leftarrow \neg e(X)$

initial:  $e(s(X)) \leftarrow \neg o(X)$   
 $e(X) \leftarrow e(X)$



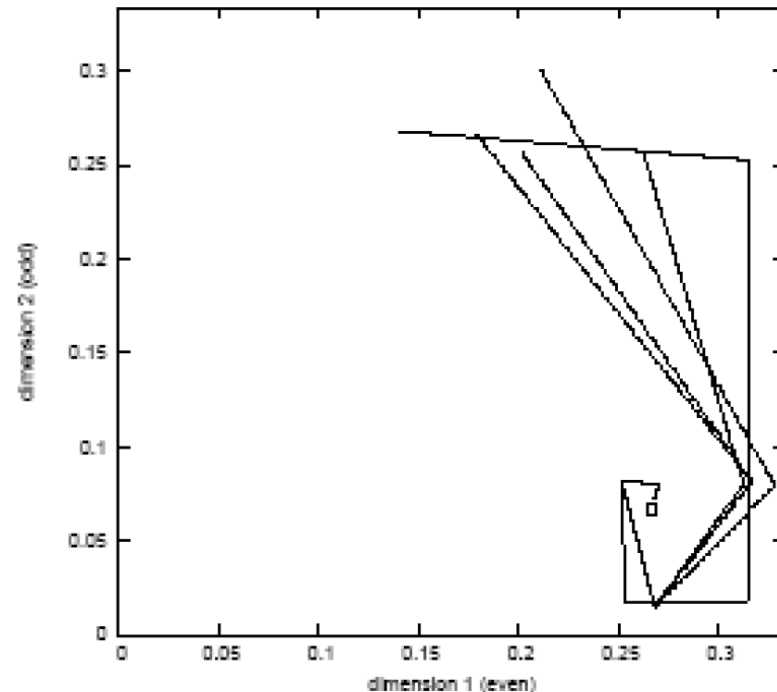
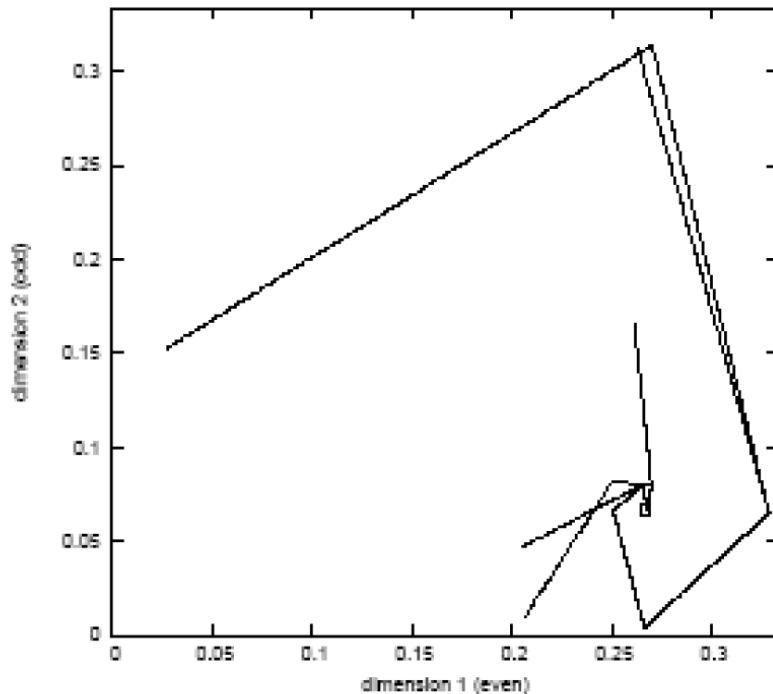
Bader, Hitzler, Hölldobler, Witzel, IJCAI-07

# Logic in Real Space



We observe convergence to unique supported model of the program.

Bader, Hitzler, Hölldobler, Witzel, IJCAI-07



**But it works only for toy size problems.  
The theoretically required embedding into real numbers cannot scale.**



# Analysis



	dimension	(a)	(b)	N/A
Interrelation	integrated (a) vs. hybrid (b)	43	0	0
	neuronal (a) vs. connectionist (b)	0	43	0
	local (a) vs. distributed (b)	2	42	0
	standard (a) vs. nonstandard (b)	43	0	0
Language	symbolic (a) vs. logical (b)	21	24	0
	propositional (a) vs. first-order (b)	3	22	18
Usage	extraction (a) vs. representation (b)	6	37	3
	learning (a) vs. reasoning (b)	19	29	0

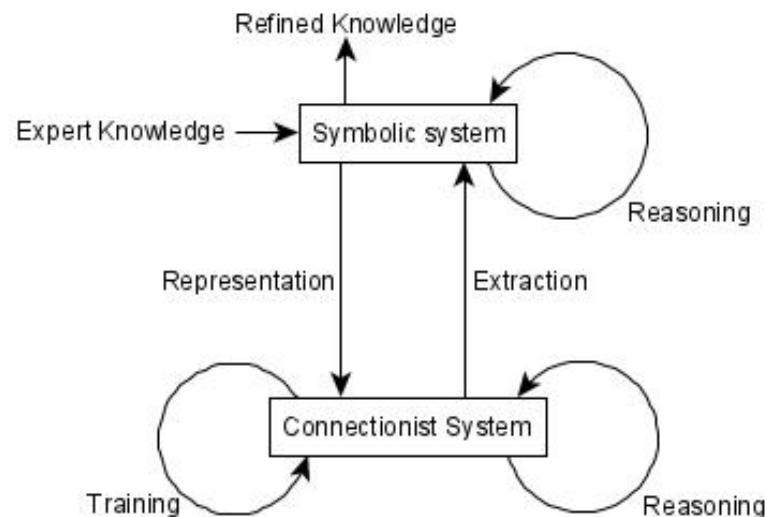


# Kautz 2020 Categories



category	number of papers
[symbolic Neuro symbolic]	13
[Symbolic[Neuro]]	9
[Neuro $\cup$ compile(Symbolic)]	10
[Neuro $\rightarrow$ Symbolic]	13
[Neuro[Symbolic]]	0

(6) We finally come to the approach to neuro-symbolic reasoning that I believe has the greatest potential to combine the strengths of logic-based and neural-based AI, namely the **Neuro[Symbolic]** architecture (Figure 15). The basic idea is to embed a symbolic reasoning engine inside a neural engine, with the goal of enabling super-neuro and combinatorial reasoning. The architecture is based on Daniel Kahneman's theory of "thinking fast and



# Deep Deductive Reasoners

Monireh Ebrahimi, Aaron Eberhart, Federico Bianchi, Pascal Hitzler,  
Towards Bridging the Neuro-Symbolic Gap: Deep Deductive Reasoners.  
Applied Intelligence 51 (9), 6326-6348, 2021.

Pascal Hitzler, Frank van Harmelen  
A reasonable Semantic Web.  
Semantic Web 1 (1-2), 39-44, 2010.

# Deep Deductive Reasoners



- We trained deep learning systems to do deductive reasoning.
- Why is this interesting?
  - For dealing with **noisy data** (where symbolic reasoners do very poorly).
  - For **speed**, as symbolic algorithms are of very high complexity.
  - Out of **principle** because we want to learn about the capabilities of deep learning for complicated cognitive tasks.
  - To perhaps begin to understand how our (neural) brains can learn to do highly symbolic tasks like formal logical reasoning, or in more generality, mathematics.  
A fundamental quest in **Cognitive Science**.

# Reasoning as Classification



- **Given a set of logical formulas (a theory).**
- **Any formula expressible over the same language is either**
  - a logical consequence or
  - not a logical consequence.
- **This can be understood as a **classification problem** for machine learning.**
- **It turns out to be a really hard machine learning problem.**

# Knowledge Materialization



- Given a set of logical formulas (a theory).
- Produce all logical consequences **under certain constraints**.
- Without **the qualifier** this is in general not possible as the set of all logical consequences is infinite.
- So we have to **constrain** to consequences of, e.g., a certain syntactic form. For relatively simple logics, this is often reasonably possible.

# Published deep deductive reasoning work

paper	logic	transfer	generative	scale	performance
[12]	RDFS	yes	no	moderate	high
[25]	RDFS	no	yes	low	high
[10]	$\mathcal{EL}^+$	no	yes	moderate	low
[20]	OWL RL	no*	no	low	high
[6]	FOL	no	yes	very low	high
(new)	RDFS	yes	yes	moderate	high?
(new)	EL+	yes	yes	moderate	high?



[12]: Ebrahimi, Sarker, Bianchi, Xie, Eberhart, Doran, Kim, **Hitzler**,  
AAAI-MAKE 2021

[25]: Makni, Hendler, SWJ 2019

[10]: Eberhart, Ebrahimi, Zhou, Shimizu, **Hitzler**, AAAI-MAKE 2020

[20]: Hohenecker, Lukasiewicz, JAIR 2020

[6]: Bianchi, **Hitzler**, AAAI-MAKE 2019

(new): Ebrahimi, Eberhart, **Hitzler** (preliminary report)

# RDFS Reasoning using Memory Networks

Monireh Ebrahimi, Md Kamruzzaman Sarker, Federico Bianchi, Ning Xie, Aaron Eberhart, Derek Doran, Hyeongsik Kim, Pascal Hitzler, Neuro-Symbolic Deductive Reasoning for Cross-Knowledge Graph Entailment. In: Proc. AAAI-MAKE 2021.

additional analysis by Sulogna Chowdhury, Aaron Eberhart and Brayden Pankaskie

# RDF reasoning



- [Note: RDF is one of the simplest useful knowledge representation languages that is not propositional.]
- Think knowledge graph.
- Think node-edge-node triples such as
  - BarackObama rdf:type President
  - BarackObama husbandOf MichelleObama
  - President rdfs:subClassOf Human
  - husbandOf rdfs:subPropertyOf spouseOf
- Then there is a (fixed, small) set of inference rules, such as  
rdf:type(x,y) AND rdfs:subClassOf(y,z) THEN rdf:type(x,z)



# RDF reasoning

- Essentially, RDF reasoning is Datalog reasoning restricted to:
  - Unary and binary predicates only.
  - A fixed set of rules that are not facts.
- You can try the following:
  - Use a vector embedding for one RDF graph.
  - Create all logical consequences.
  - Throw  $n\%$  of them away.
  - Use the rest to train a DL system.
  - Check how many of those you threw away can be recovered this way.



Semantic Web – Interoperability, Usability, Applicability an IOS Press Journal

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**Deep Learning for Noise-Tolerant RDFS Reasoning**

Submitted by Bassem Makni on 10/01/2018 - 01.02

Tracking #: 2028-3241

**A new version of this paper is available**

**Authors:**  
Bassem Makni  
James Hendler

**Responsible editor:**  
Guest Editors Semantic Deep Learning 2018

**Submission type:**  
Full Paper

**Abstract:**  
Since the 2001 envisioning of the Semantic Web (SW) [1] as an extension to the World Wide Web, the main research focus in SW

# RDF reasoning



- **The problem with the approach just described:**
  - It works only with that graph.
- **What you'd really like to do is:**
  - Train a deep learning system so that you can present a new, unseen graph to it, and it can correctly derive the deductively inferred triples.
- **Note:**
  - You don't know the IRIs in the graph up front. The only overlap may or may not be the IRIs in the rdf/s namespace.
  - You don't know up front how "deep" the reasoning needs to be.
  - There is no lack of training data, it can be auto-generated.

# Representation

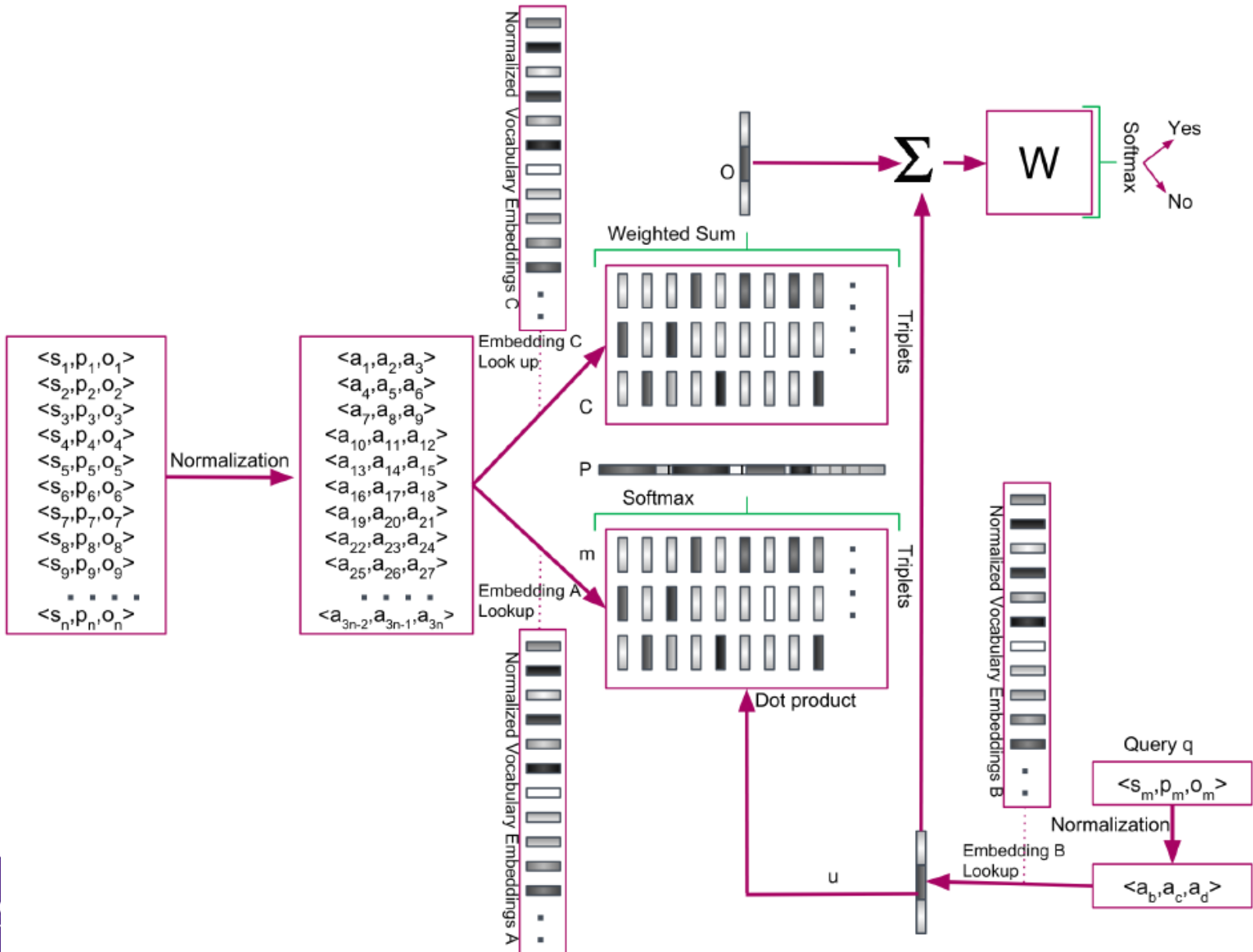
- **Goal is to be able to reason over unseen knowledge graphs. I.e. the out-of-vocabulary problem needs addressing.**
- **Normalization of vocabulary (i.e., it becomes shared vocabulary across all input knowledge graphs.**
- **One vocabulary item becomes a one-hot vector (dimension  $d$ , number of normalized vocabulary terms)**
- **One triple becomes a  $3 \times d$  matrix.**
- **The knowledge graph becomes an  $n \times 3 \times d$  tensor ( $n$  is the number of knowledge graph triples)**
- **Knowledge graph is stored in “memory”**





- **An attention mechanism retrieves memory slots useful for finding the correct answer to a query.**
- **These are combined with the query and run through a (learned) matrix to retrieve a new (processed) query.**
- **This is repeated (in our experiment with 10 “hops”).**
- **The final out put is a yes/no answer to the query.**

# Memory Network based on MemN2N



# Experiments: Performance



Test Dataset	#KG	Base						Inferred						Invalid
		#Facts	#Ent.	%Class	%Indv	%R.	%Axiom.	#Facts	#Ent.	%Class	%Indv	%R.	%Axiom.	#Facts
OWL-Centric	2464	996	832	14	19	3	0	494	832	14	0.01	1	20	462
Linked Data	20527	999	787	3	22	5	0	124	787	3	0.006	1	85	124
OWL-Centric Test Set	21	622	400	36	41	3	0	837	400	36	3	1	12	476
Synthetic Data	2	752	506	52	0	1	0	126356	506	52	0	1	0.07	700

Table 2: Statistics of various datasets used in experiments

Baseline: non-normalized embeddings, same architecture

Training Dataset	Test Dataset	Valid Triples Class			Invalid Triples Class			Accuracy
		Precision	Recall /Sensitivity	F-measure	Precision	Recall /Specificity	F-measure	
OWL-Centric Dataset	Linked Data	93	98	96	98	93	95	<b>96</b>
OWL-Centric Dataset (90%)	OWL-Centric Dataset (10%)	88	91	89	90	88	89	<b>90</b>
OWL-Centric Dataset	OWL-Centric Test Set <sup>b</sup>	79	62	68	70	84	76	<b>69</b>
OWL-Centric Dataset	Synthetic Data	65	49	40	52	54	42	<b>52</b>
OWL-Centric Dataset	Linked Data <sup>a</sup>	54	98	70	91	16	27	86
OWL-Centric Dataset <sup>a</sup>	Linked Data <sup>a</sup>	62	72	67	67	56	61	91
OWL-Centric Dataset(90%) <sup>a</sup>	OWL-Centric Dataset(10%) <sup>a</sup>	79	72	75	74	81	77	80
OWL-Centric Dataset	OWL-Centric Test Set <sup>ab</sup>	58	68	62	62	50	54	58
OWL-Centric Dataset <sup>a</sup>	OWL-Centric Test Set <sup>ab</sup>	77	57	65	66	82	73	73
OWL-Centric Dataset	Synthetic Data <sup>a</sup>	70	51	40	47	52	38	51
OWL-Centric Dataset <sup>a</sup>	Synthetic Data <sup>a</sup>	67	23	25	52	80	62	50
<b>Baseline</b>								
OWL-Centric Dataset	Linked Data	73	98	83	94	46	61	43
OWL-Centric Dataset (90%)	OWL-Centric Dataset (10%)	84	83	84	84	84	84	82
OWL-Centric Dataset	OWL-Centric Test Set <sup>b</sup>	62	84	70	80	40	48	61
OWL-Centric Dataset	Synthetic Data	35	41	32	48	55	45	48

<sup>a</sup> More Tricky Nos & Balanced Dataset

<sup>b</sup> Completely Different Domain.

Table 3: Experimental results of proposed model

# Experiments: Reasoning Depth



Test Dataset	Hop 0			Hop 1			Hop 2			Hop 3			Hop 4			Hop 5			Hop 6			Hop 7			Hop 8			Hop 9			Hop 10					
	P	R	F	P	R	F	P	R	F	P	R	F	P	R	F	P	R	F	P	R	F	P	R	F	P	R	F	P	R	F	P	R	F	P	R	F
Linked Data <sup>a</sup>	0	0	0	80	99	88	89	97	93	77	98	86	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Linked Data <sup>b</sup>	2	0	0	82	91	86	89	98	93	79	100	88	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
OWL-Centric <sup>c</sup>	19	5	9	31	75	42	78	80	78	48	47	44	4	34	6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Synthetic	32	46	33	31	87	38	66	55	44	25	45	32	29	46	33	26	46	33	25	46	33	25	46	33	24	43	31	25	43	31	22	36	28			

<sup>a</sup> LemonUby Ontology

<sup>b</sup> Agrovoc Ontology

<sup>c</sup> Completely Different Domain

Table 4: Experimental results over each reasoning hop

Dataset	Hop 1	Hop 2	Hop 3	Hop 4	Hop 5	Hop 6	Hop 7	Hop 8	Hop 9	Hop 10
<i>OWL-Centric</i> <sup>a</sup>	8%	67%	24%	0.01%	0%	0%	0%	0%	0%	0%
Linked Data <sup>b</sup>	31%	50%	19%	0%	0%	0%	0%	0%	0%	0%
Linked Data <sup>c</sup>	34%	46%	20%	0%	0%	0%	0%	0%	0%	0%
OWL-Centric <sup>d</sup>	5%	64%	30%	1%	0%	0%	0%	0%	0%	0%
Synthetic Data	0.03%	1.42%	1%	1.56%	3.09%	6.03%	11.46%	20.48%	31.25%	23.65%

<sup>a</sup> Training Set

<sup>b</sup> LemonUby Ontology

<sup>c</sup> Agrovoc Ontology

<sup>d</sup> Completely Different Domain

Table 5: Data distribution per knowledge graph over each reasoning hop

Training time: just over a full day

# Generative RDFS Reasoning using Pointer Networks

Monireh Ebrahimi, Aaron Eberhart, Pascal Hitzler

On the Capabilities of Pointer Networks for Deep Deductive Reasoning

<https://arxiv.org/abs/2106.09225>

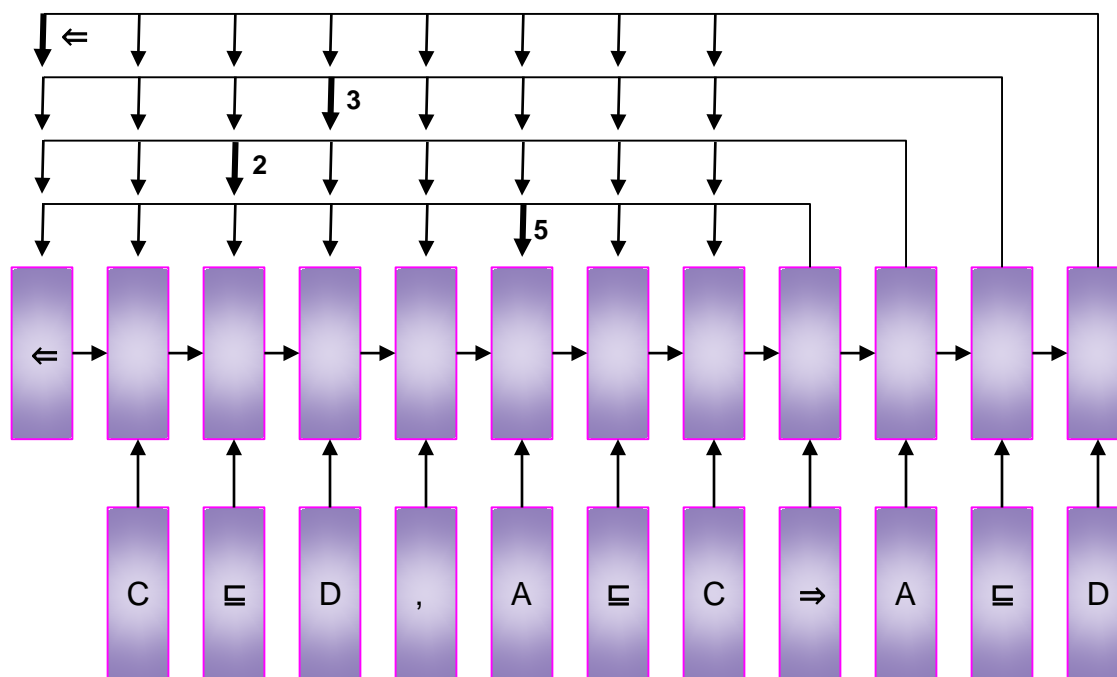


- **Pointer Networks ‘point’ to input elements!**
- **Ptr-Net approach specifically targets problems whose outputs are discrete and correspond to positions in the input.**
- **At each time step, the distribution of the attention is the answer!**
- **Application:**
  - **NP-hard Travelling Salesman Problem (TSP)**
  - **Delaunay Triangulation**
  - **Convex Hull**
  - **Text Summarization**
  - **Code completion**
  - **Dependency Parsing**

# Pointer Networks for Reasoning



- To mimic human reasoning behaviour where one can learn to choose a set of symbols in different locations and copy these symbols to suitable locations to generate new logical consequences based on a set of predefined logical entailment rules



$$C \subseteq D, A \subseteq C \mapsto A \subseteq D$$



# Results without transfer

Logic	KG Size	Pointer Networks		Transformer			LSTM
		SubWordText	Tokenizer	Normalized	Not-Normalized		
					SubWordText	Tokenizer	
RDF	3 - 735	87%	<b>99%</b>	5%	25%	4%	0.17%

- On RDF, slightly outperforms [Hendler Makni SWJ 2019] approach.
- Our approach is a more straightforward application.
- Evaluation is on the same dataset.

# Results with transfer



**Table 6** Exact Match Accuracy Results for Transfer Learning/Representation: SubWord-Text Tokenization Encoding

Train \ Test	LUBM	Awards	University
LUBM	*	75%	78%
Awards	79%	*	77%
University	81%	82%	*

**Table 7** Exact Match Accuracy Results for Transfer Learning/Representation: Whitespace Tokenization Encoding

Train \ Test	LUBM	Awards	University
LUBM	*	61%	47%
Awards	96%	*	84%
University	82%	88%	*

# Completion Reasoning Emulation for the Description Logic EL+

Aaron Eberhart, Monireh Ebrahimi, Lu Zhou, Cogan Shimizu, Pascal Hitzler,  
Completion Reasoning Emulation for the Description Logic EL+.  
In: Andreas Martin, Knut Hinkelmann, Hans-Georg Fill, AURORA Gerber, Doug  
Lenat, Reinhard Stolle, Frank van Harmelen (eds.), Proceedings of the  
AAAI 2020 Spring Symposium on Combining Machine Learning and Knowledge  
Engineering in Practice, AAAI-MAKE 2020, Palo Alto, CA, USA, March 23-25,  
2020, Volume I.

# EL+ is essentially OWL 2 EL



Table 2:  $\mathcal{EL}^+$  Completion Rules

$CX \sqsubseteq CY$
$CX \sqcap CY \sqsubseteq CZ$
$CX \sqsubseteq \exists RY.CZ$
$\exists RX.CY \sqsubseteq CZ$
$RX \sqsubseteq RY$
$RX \circ RY \sqsubseteq RZ$

(1)	$A \sqsubseteq C$	$C \sqsubseteq D$	$\models A \sqsubseteq D$
(2)	$A \sqsubseteq C_1$	$A \sqsubseteq C_2$	$C_1 \sqcap C_2 \sqsubseteq D \models A \sqsubseteq D$
(3)	$A \sqsubseteq C$	$C \sqsubseteq \exists R.D$	$\models A \sqsubseteq \exists R.D$
(4)	$A \sqsubseteq \exists R.B$	$B \sqsubseteq C$	$\exists R.C \sqsubseteq D \models A \sqsubseteq D$
(5)	$A \sqsubseteq \exists S.D$	$S \sqsubseteq R$	$\models A \sqsubseteq \exists R.D$
(6)	$A \sqsubseteq \exists R_1.C$	$C \sqsubseteq \exists R_2.D$	$R_1 \circ R_2 \sqsubseteq R \models A \sqsubseteq \exists R.D$

Table 1:  $\mathcal{EL}^+$  Semantics

Description	Expression	Semantics
Individual	$a$	$a \in \Delta^{\mathcal{I}}$
Top	$\top$	$\Delta^{\mathcal{I}}$
Bottom	$\perp$	$\emptyset$
Concept	$C$	$C^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
Role	$R$	$R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
Conjunction	$C \sqcap D$	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$
Existential Restriction	$\exists R.C$	$\{ a \mid \text{there is } b \in \Delta^{\mathcal{I}} \text{ such that } (a, b) \in R^{\mathcal{I}} \text{ and } b \in C^{\mathcal{I}} \}$
Concept Subsumption	$C \sqsubseteq D$	$C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
Role Subsumption	$R \sqsubseteq S$	$R^{\mathcal{I}} \subseteq S^{\mathcal{I}}$
Role Chain	$R_1 \circ \dots \circ R_n \sqsubseteq R$	$R_1^{\mathcal{I}} \circ \dots \circ R_n^{\mathcal{I}} \subseteq R^{\mathcal{I}}$

with  $\circ$  signifying standard binary composition

Table 7: Average Precision Recall and F1-score For each Distance Evaluation

	Atomic Levenshtein Distance			Character Levenshtein Distance			Predicate Distance		
	Precision	Recall	F1-score	Precision	Recall	F1-score	Precision	Recall	F1-score
	Synthetic Data								
Piecewise Prediction	0.138663	0.142208	0.140412	0.138663	0.142208	0.140412	0.138646	0.141923	0.140264
Deep Prediction	<b>0.154398</b>	<b>0.156056</b>	<b>0.155222</b>	<b>0.154398</b>	<b>0.156056</b>	<b>0.155222</b>	<b>0.154258</b>	<b>0.155736</b>	<b>0.154993</b>
Flat Prediction	0.140410	0.142976	0.141681	0.140410	0.142976	0.141681	0.140375	0.142687	0.141521
Random Prediction	0.010951	0.0200518	0.014166	0.006833	0.012401	0.008811	0.004352	0.007908	0.007908
	SNOMED Data								
Piecewise Prediction	0.010530	0.013554	0.011845	0.010530	0.013554	0.011845	0.010521	0.013554	0.011839
Deep Prediction	<b>0.015983</b>	0.0172811	<b>0.016595</b>	<b>0.015983</b>	0.017281	<b>0.016595</b>	<b>0.015614</b>	0.017281	<b>0.016396</b>
Flat Prediction	0.014414	<b>0.018300</b>	0.016112	0.0144140	<b>0.018300</b>	0.016112	0.013495	<b>0.018300</b>	0.015525
Random Prediction	0.002807	0.006803	0.003975	0.001433	0.003444	0.002023	0.001769	0.004281	0.002504

# Generative EL Reasoning using Pointer Networks

Monireh Ebrahimi, Aaron Eberhart, Pascal Hitzler  
On the Capabilities of Pointer Networks for Deep Deductive Reasoning  
<https://arxiv.org/abs/2106.09225>



# Results with transfer



Logic	KG Size	Pointer Networks		Transformer			LSTM
		SubWordText	Tokenizer	Normalized	Not-Normalized		
					SubWordText	Tokenizer	
ER	40	73%	<b>73%</b>	8%	8%	0.4 %	0%
	50	68%	<b>68%</b>	11%	11%	0.3%	0%
	120	49%	<b>49%</b>	15%	NA	NA	0%

- same architecture as before



# Conclusions

# Conclusions



- **Bridging the neuro-symbolic gap is still a major quest.**
- **Research on Deep Deductive Reasoning is at the heart of neuro-symbolic Artificial Intelligence**
  - **Research is needed to push the envelope with respect to core aspects such as**
  - **more complex logics**
  - **higher reasoning accuracy**
  - **better transfer**
  - **scalability**



**Thanks!**

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**Thanks!**