

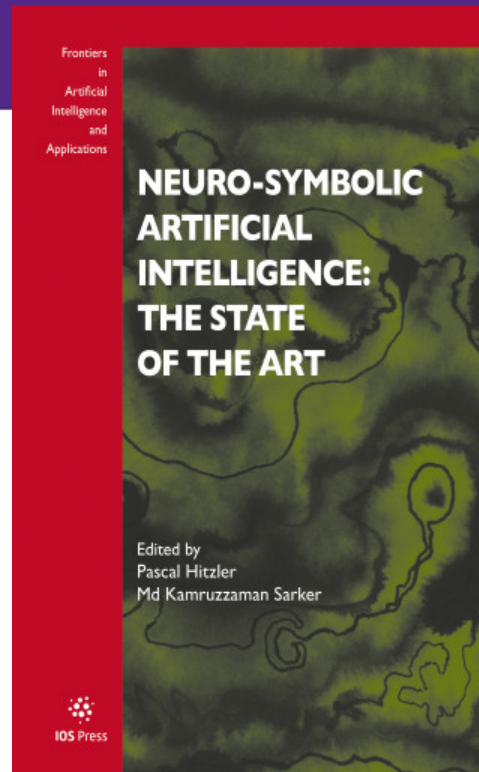
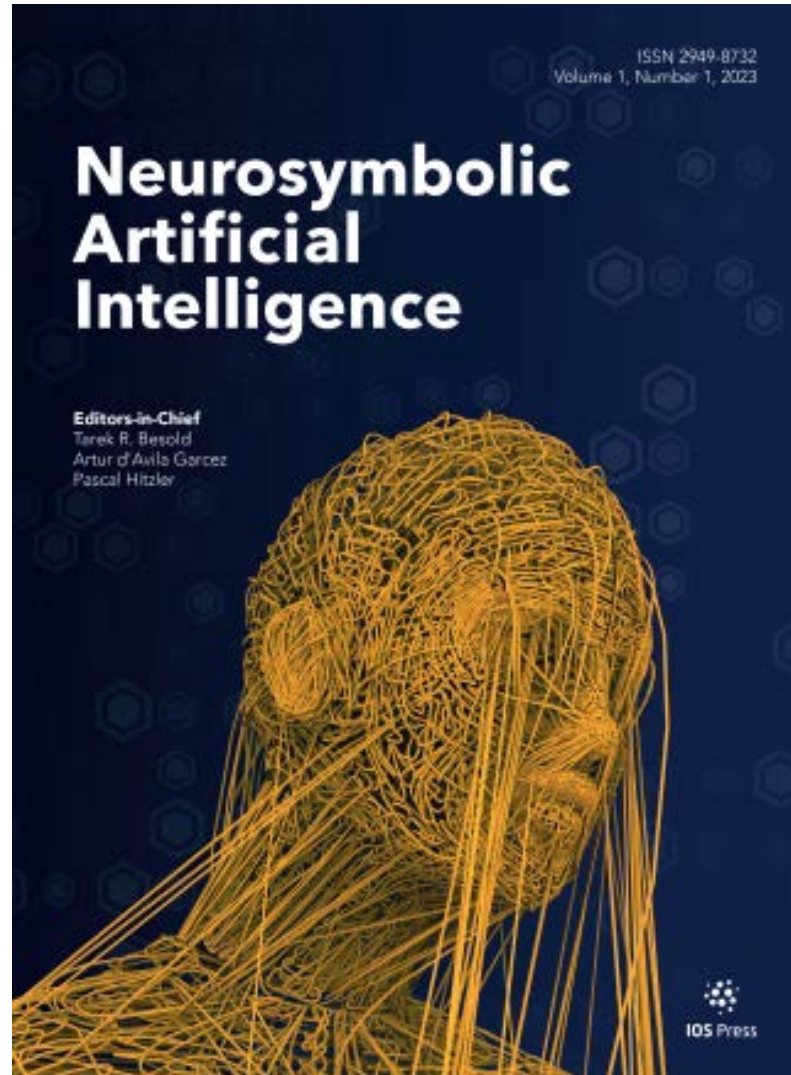
# Neurosymbolic AI – a selective history



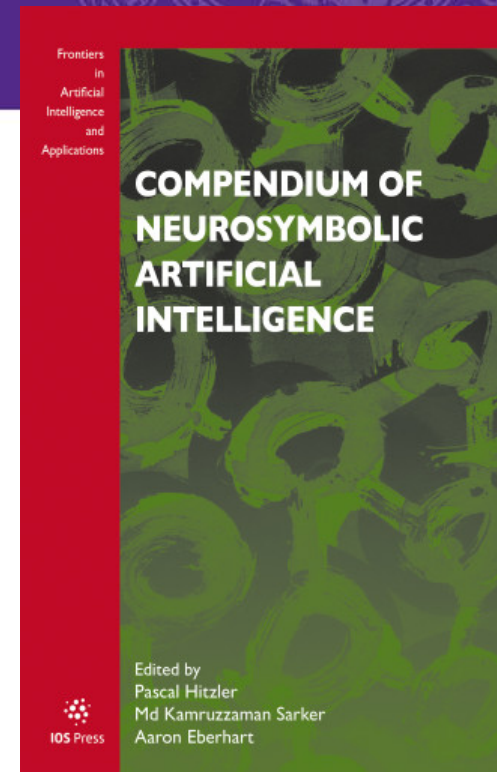
## Pascal Hitzler

Data Semantics Laboratory (DaSe Lab)  
Kansas State University

<http://www.daselab.org>



2022, 17 chapters



2023, 30 chapters

**Neurosymbolic AI community slack currently over 800 members email [hitzler@ksu.edu](mailto:hitzler@ksu.edu) to get an invite**

# Neuro-symbolic AI



**Publications on neuro-symbolic AI in major conferences  
(research papers only):**

conference	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	total
ICML	0	0	0	0	0	1	3	2	5	6	17
NeurIPS	0	0	0	0	0	0	0	4	2	4	10
AAAI	0	0	0	0	0	1	0	1	1	1	4
IJCAI	1	0	0	0	0	0	2	2	0	2	7
ICLR	N/A	N/A	0	0	0	0	1	1	1	3	6
total	1	0	0	0	0	2	6	10	9	16	44

**See**

**Md Kamruzzaman Sarker, Lu Zhou, Aaron Eberhart, Pascal Hitzler  
Neuro-Symbolic Artificial Integration: Current Trends  
AI Communications 34 (3), 197-209, 2022.**

**Neurosymbolic? Neuro-symbolic?  
Neural-Symbolic? Symbolic-Subsymbolic?**

# Neural



- Refers to computational abstractions of (natural) neural network systems.
- Prominently includes Artificial Neural Networks and Deep Learning as machine learning paradigms.
- More generally sometimes referred to as *connectionist systems*.
  
- Prominent applications come from the machine learning world.
- And of course, there is the current deep learning hype.

# Symbolic



- Refers to (computational) symbol manipulations of all kind.
- Graphs and trees, traversal, data structure operations.
- Knowledge representation in explicit symbolic form (data base, ontology, knowledge graph)
- Inductive and statistical inference.
- Formal logical (deductive or abductive) reasoning.
- Prominent applications all over computer science, including expert systems (and their modern versions), information systems, data management, added value of data annotation, etc.
- Semantic Web data is inherently symbolic.

## Computer Science perspective:

- **Connectionist machine learning systems are**
  - very powerful for some machine learning problems
  - robust to data noise
  - very hard to understand or explain
  - really poor at symbol manipulation
  - unclear how to effectively use background (domain) knowledge
- **Symbolic systems are**
  - Usually rather poor regarding machine learning problems
  - Intolerant to data noise
  - Relatively easy to analyse and understand
  - Really good at symbol manipulation
  - Designed to work with other (background) knowledge





## Computer Science perspective:

- **Let's try to get the best of both worlds:**
  - very powerful machine learning paradigm
  - robust to data noise
  - easy to understand and assess by humans
  - good at symbol manipulation
  - work seamlessly with background (domain) knowledge
  
- **How to do that?**
  - Endow connectionist systems with symbolic components?
  - Add connectionist learning to symbolic reasoners?





## Note:

- **Deep Learning systems are a far cry from how natural neural networks work.**
- **There are things that our brain can do, and things that symbolic approaches can do, where we do not have the faintest idea how to solve them through deep learning (or any other connectionist learning approach).**
- **The argument that we “just don’t have enough training data” speaks (understandably) to the current hype, but is a hope that is unfounded: While this may be the case in some cases, there is no rationale to overgeneralize.  
[Note: if we had “enough computational power,” we could also solve all reasonable-size NP-complete problems in an instant.]**

# The Interface Issue



- **Symbolic knowledge comes as logical theories (sets of formulas over a logic)**
- **Subsymbolic systems process tuples of real/float numbers (vectors, matrices, tensors)**
- **How do you interface?**
- **How do you map between the symbolic world and the subsymbolic world?**

**Some key problems that need to be overcome:**

- **Logic is full of highly structured objects, how to represent them in Real Space?**
- **How to represent variable bindings in a distributed setting?**
- **The required length of logical deduction chain is not known up front.**



# Representation Issues

# McCulloch & Pitts, 1943



- McCulloch & Pitts 1943
  - Neurons with binary activation functions.
  - Modelling of propositional connectives.
  - Networks equivalent to finite automata.

Values 0 („false“) and 1 („true“) being propagated.

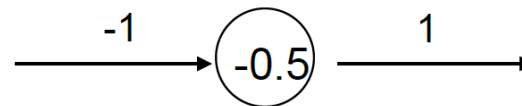


disjunction

Simultaneous update of all nodes in network.



conjunction



negation

# McCulloch & Pitts follow-on



- Hölldobler & Kalinke 1994
  - Extends the approach by McCulloch & Pitts.
  - Representation of propositional logic programs and their semantics.
  - „Massively parallel reasoning.“

logic program

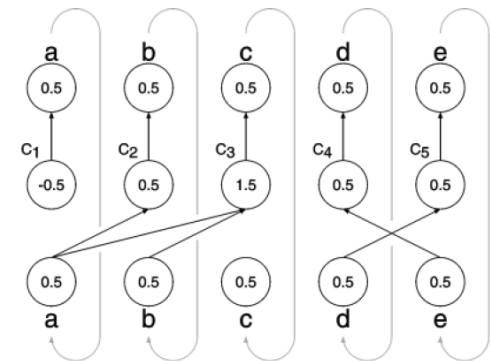
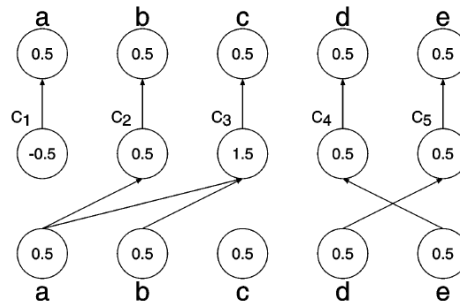


core net



recurrent net

$a \leftarrow$   
 $b \leftarrow a$   
 $c \leftarrow a \wedge b$   
 $d \leftarrow e$   
 $e \leftarrow d$



# McCulloch & Pitts follow-on

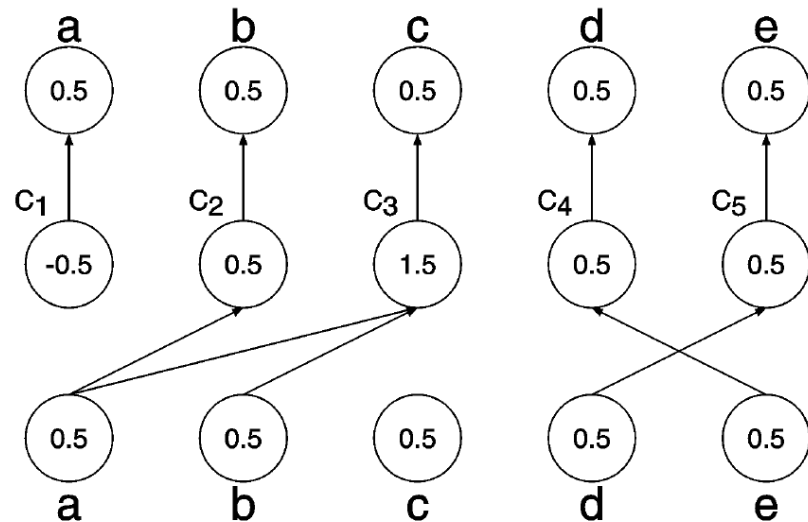


Logic program P



core net

$a \leftarrow$   
 $b \leftarrow a$   
 $c \leftarrow a \wedge b$   
 $d \leftarrow e$   
 $e \leftarrow d$



- Update „along implication“.
- Corresponds to computing the semantic operator  $T_P$ .
- $T_P$  represents meaning (semantics) of P through its fixed points.

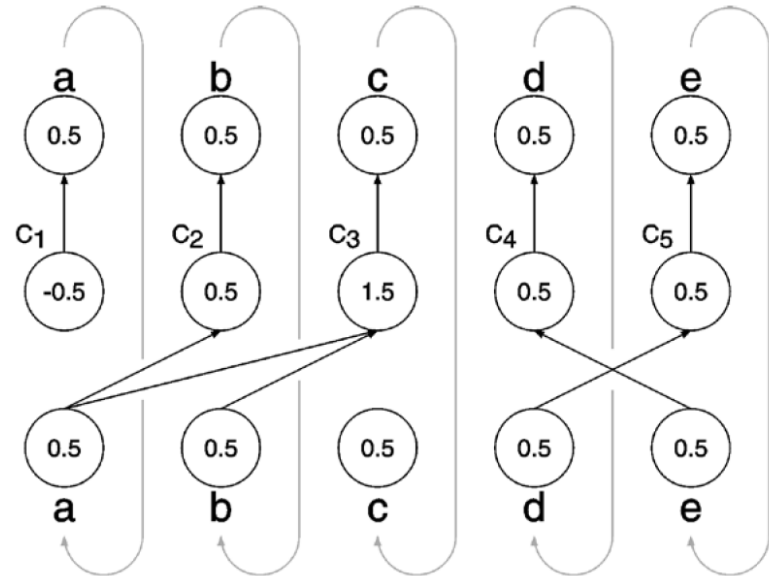
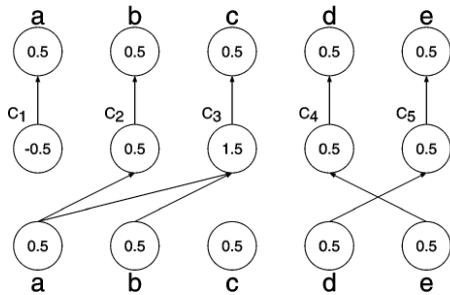
# McCulloch & Pitts follow-on



core net



recurrent net

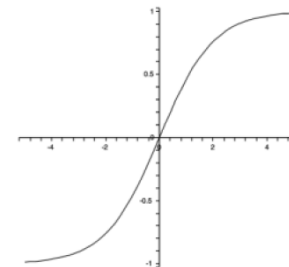
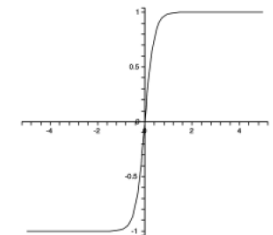
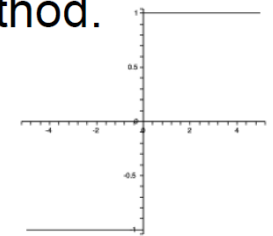


- Repeated updates along layers corresponds to iterations of the semantic operator.
- Semantics of the program (= fixed point of the operator) can be computed in a parallel manner.

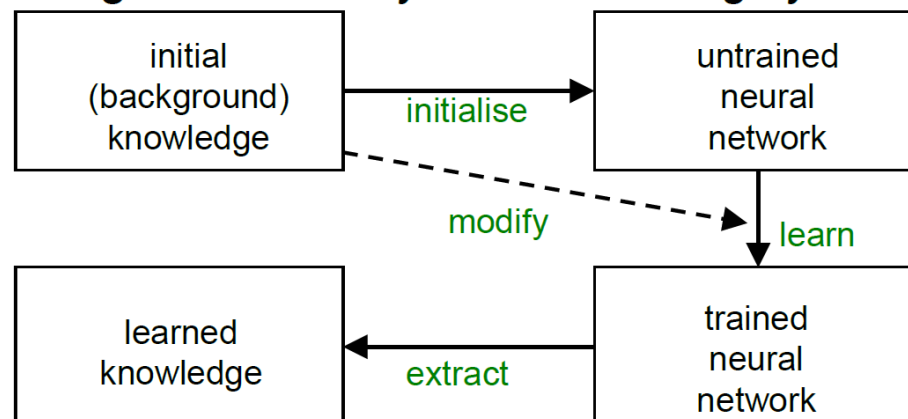


# McCulloch & Pitts follow-on

- Garcez & Zaverucha 1999  
Garcez, Broda & Gabbay 2001
- Development of a learning paradigm from the Core Method.
- Required: differentiable activation function.
  - Allows learning with standard methods.
  - Backpropagation algorithm.



- Establishing the *neural-symbolic learning cycle*.



# The catch



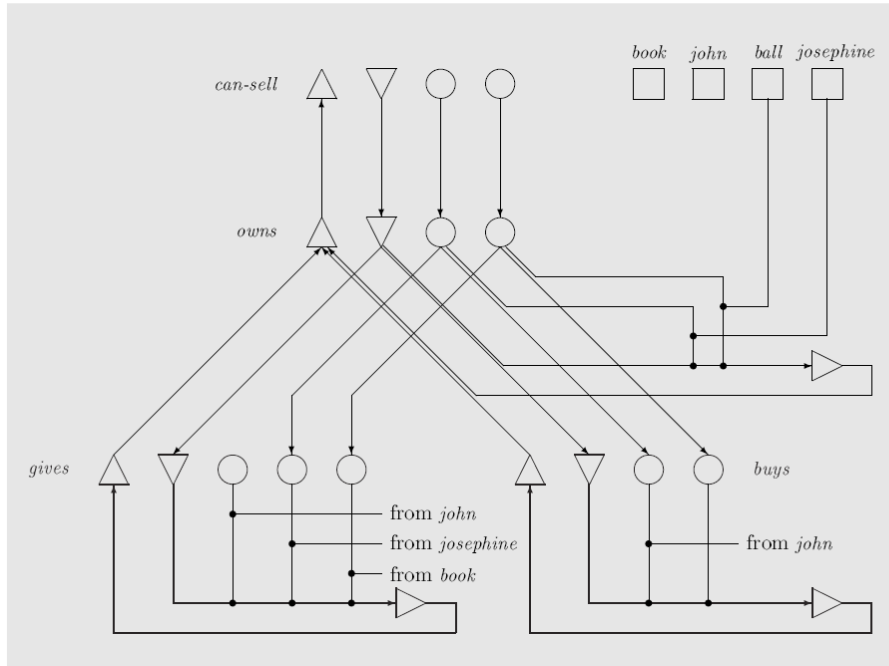
- **This is all propositional.**
- **There's only that much you can do with propositional logic.**
- **In particular, in terms of knowledge representation and reasoning, propositional logic doesn't really get you anything useful.**

**E.g.**

- **RDF (knowledge graphs) is already much closer to datalog than to propositional logic.**
- **OWL (knowledge graph schemas) is a fragment of first-order predicate logic.**

# Variable Binding

## SHRUTI



Shastri & Ajjanagadde 1993

Variable binding  
via time synchronization.

*Reflexive* (i.e. fast)  
*reasoning* possible.

Picture: Hölldobler,  
*Introduction to  
Computational Logic*, 2001

$\text{gives}(X,Y,Z) \rightarrow \text{owns}(Y,Z)$

$\text{buys}(X,Y) \rightarrow \text{owns}(X,Y)$

$\text{owns}(X,Y) \rightarrow \text{can-sell}(X,Y)$

$\text{gives}(\text{john}, \text{josephine}, \text{book})$

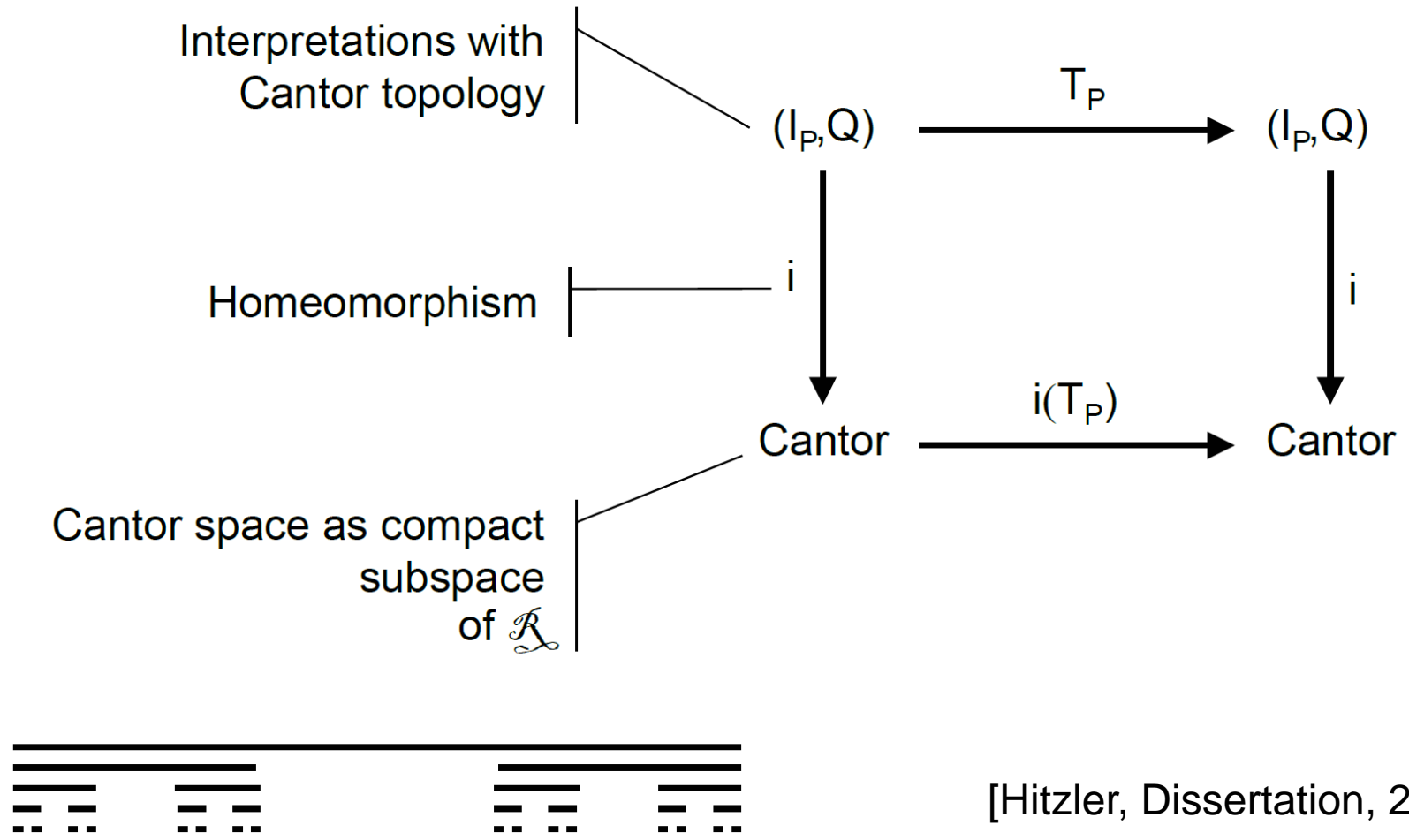
$(\exists X) \text{buys}(\text{john}, X)$

$\text{owns}(\text{josephine}, \text{ball})$

### Problems:

- It's still essentially datalog.
- It has a globally bounded reasoning depth.
- \* It doesn't really learn.

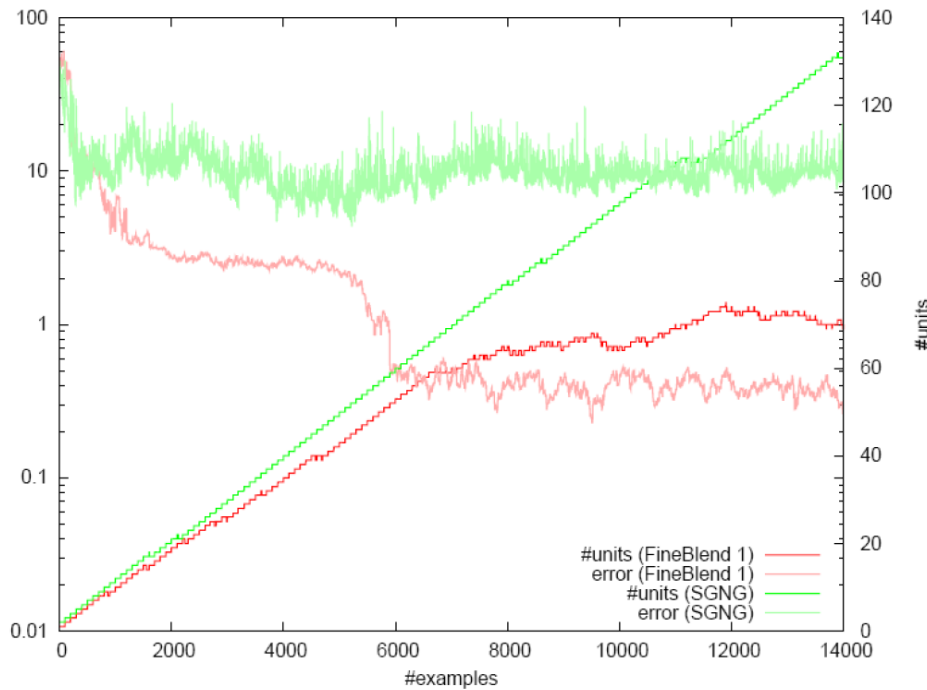
# Logic in Real Space



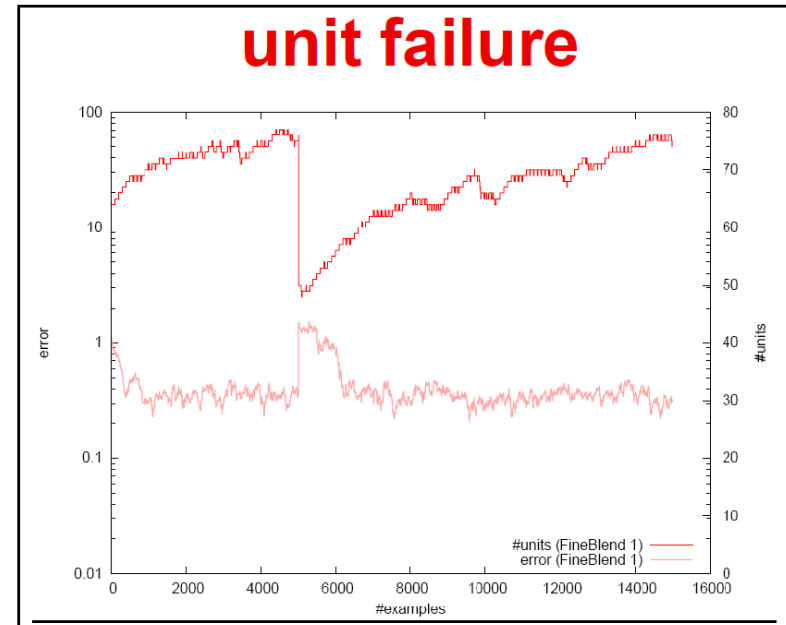
[Hitzler, Dissertation, 2001]

# Logic in Real Space

Bader, Hitzler, Hölldobler,  
Witzel, IJCAI-07



**Architecture is mix of radial basis function network and neural gas approach.**



target:  $e(0).$   
 $e(s(X)) \leftarrow o(X).$   
 $o(X) \leftarrow \neg e(X)$

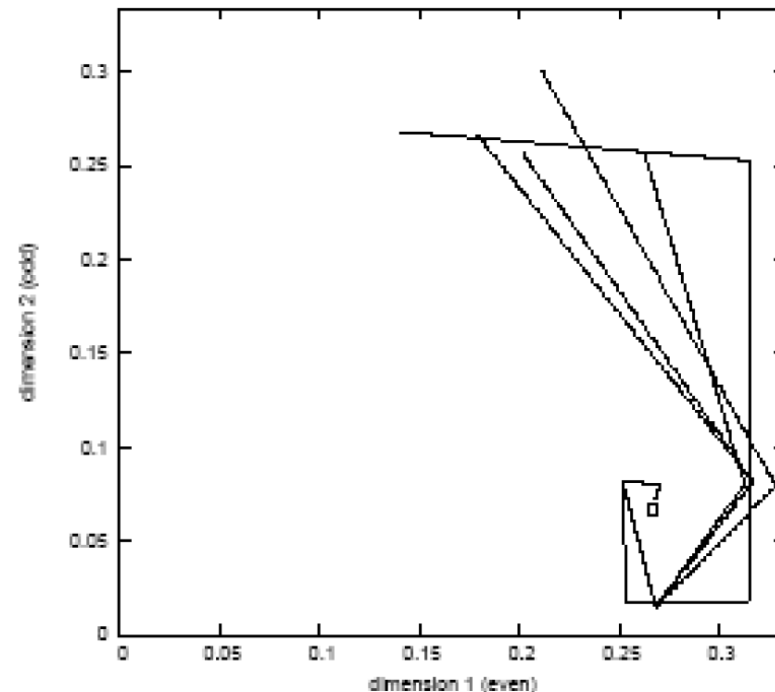
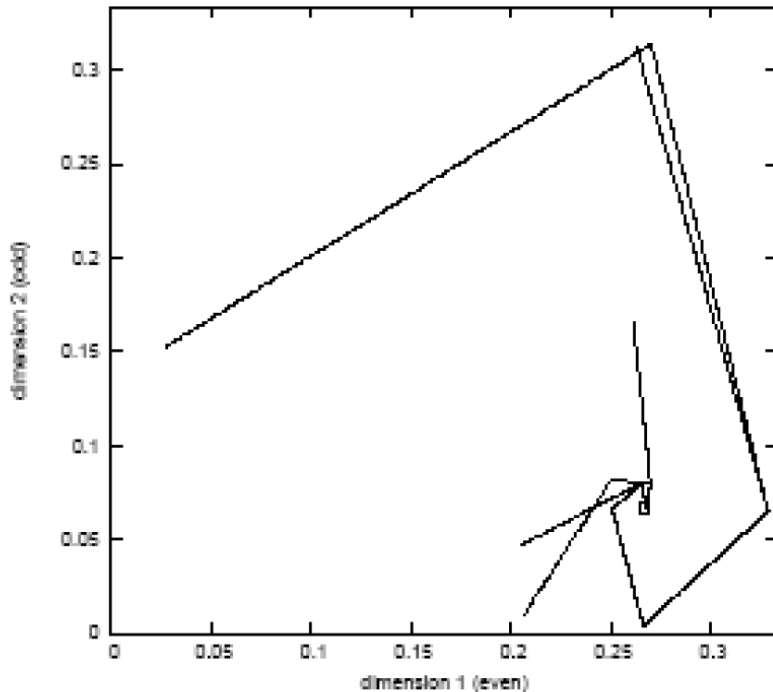
initial:  $e(s(X)) \leftarrow \neg o(X)$   
 $e(X) \leftarrow e(X)$

# Logic in Real Space



We observe convergence to unique supported model of the program.

Bader, Hitzler, Hölldobler, Witzel, IJCAI-07

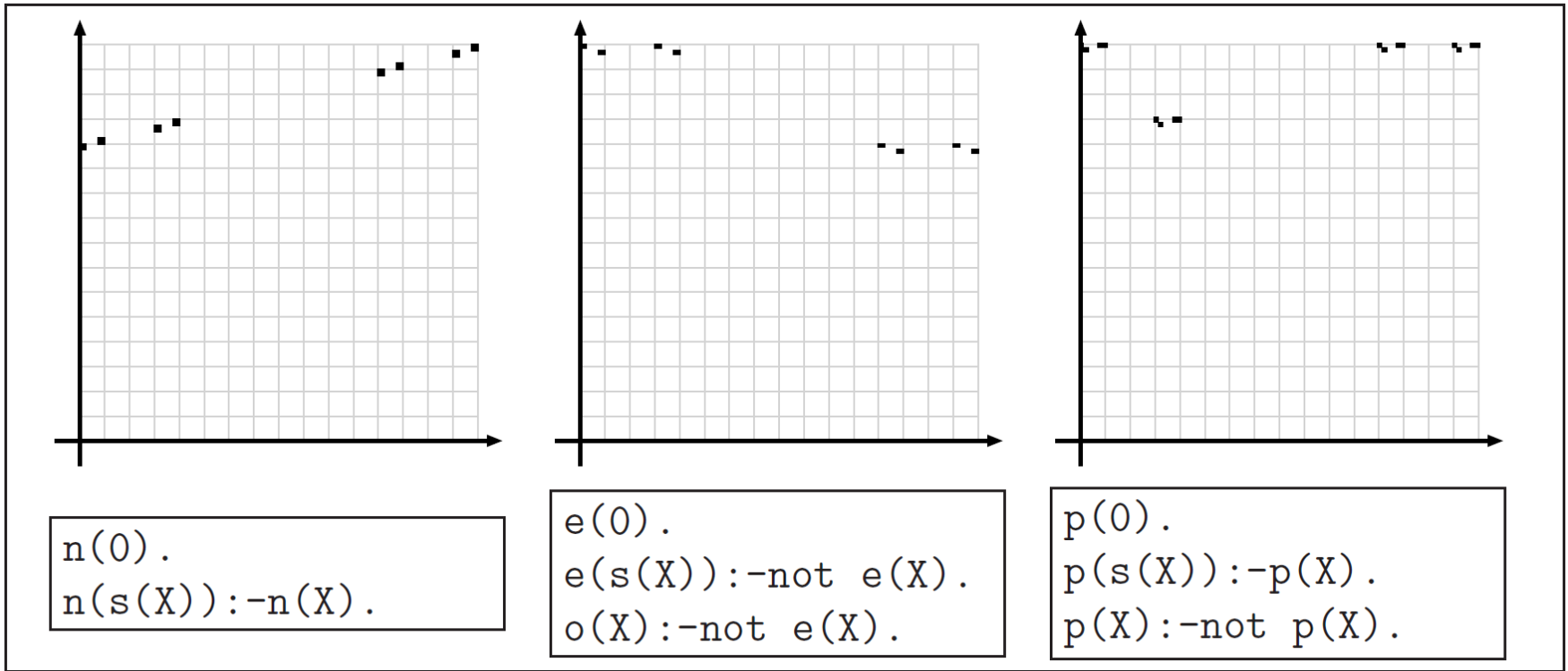


**But it works only for toy size problems.  
The theoretically required embedding into real numbers cannot scale.**



# Some other early contributions





**Consequence operators of logic programs mapped to the Cantor set in the real numbers are fractals (self-similar) – and can formally described by means of Iterated Function Systems.**

# Rule Extraction



**[Lehmann, Bader, Hitzler 2010; Labaf, Hitzler, Evans 2017]**

- **There is always a unique reduced/minimal (definite) propositional logic program that captures the input-output behavior of a given ANN (3-layer feedforward neural network with threshold activation functions).**
- **Corresponding programs always exist and are usually smaller if additional background knowledge is taken into account, but uniqueness is not guaranteed then.**



# Current Trends

# Neuro-symbolic AI



## Publications on neuro-symbolic AI in major conferences (research papers only):

conference	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	total
ICML	0	0	0	0	0	1	3	2	5	6	17
NeurIPS	0	0	0	0	0	0	0	4	2	4	10
AAAI	0	0	0	0	0	1	0	1	1	1	4
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See

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**Neuro-Symbolic Artificial Integration: Current Trends**  
**AI Communications 34 (3), 197-209, 2022.**

# Comparison with 2005 survey [Bader, Hitzler 2005]



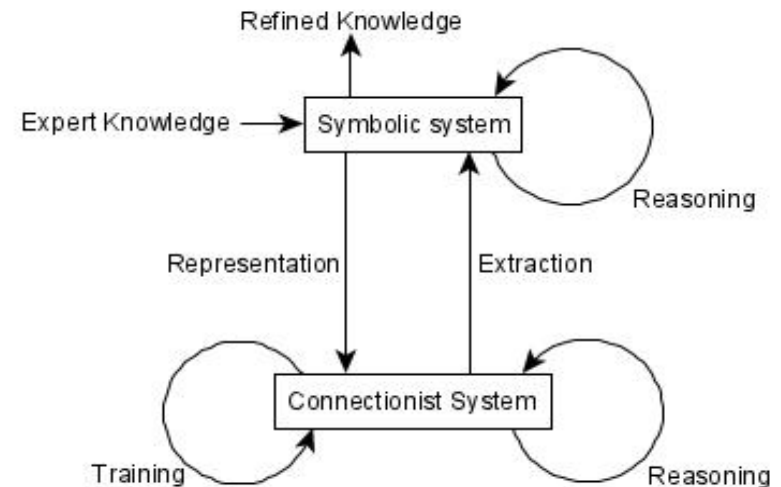
	dimension	(a)	(b)	N/A
Interrelation	integrated (a) vs. hybrid (b)	43	0	0
	neuronal (a) vs. connectionist (b)	0	43	0
	local (a) vs. distributed (b)	2	42	0
	standard (a) vs. nonstandard (b)	43	0	0
Language	symbolic (a) vs. logical (b)	21	24	0
	propositional (a) vs. first-order (b)	3	22	18
Usage	extraction (a) vs. representation (b)	6	37	3
	learning (a) vs. reasoning (b)	19	29	0

# Kautz 2020 Categories

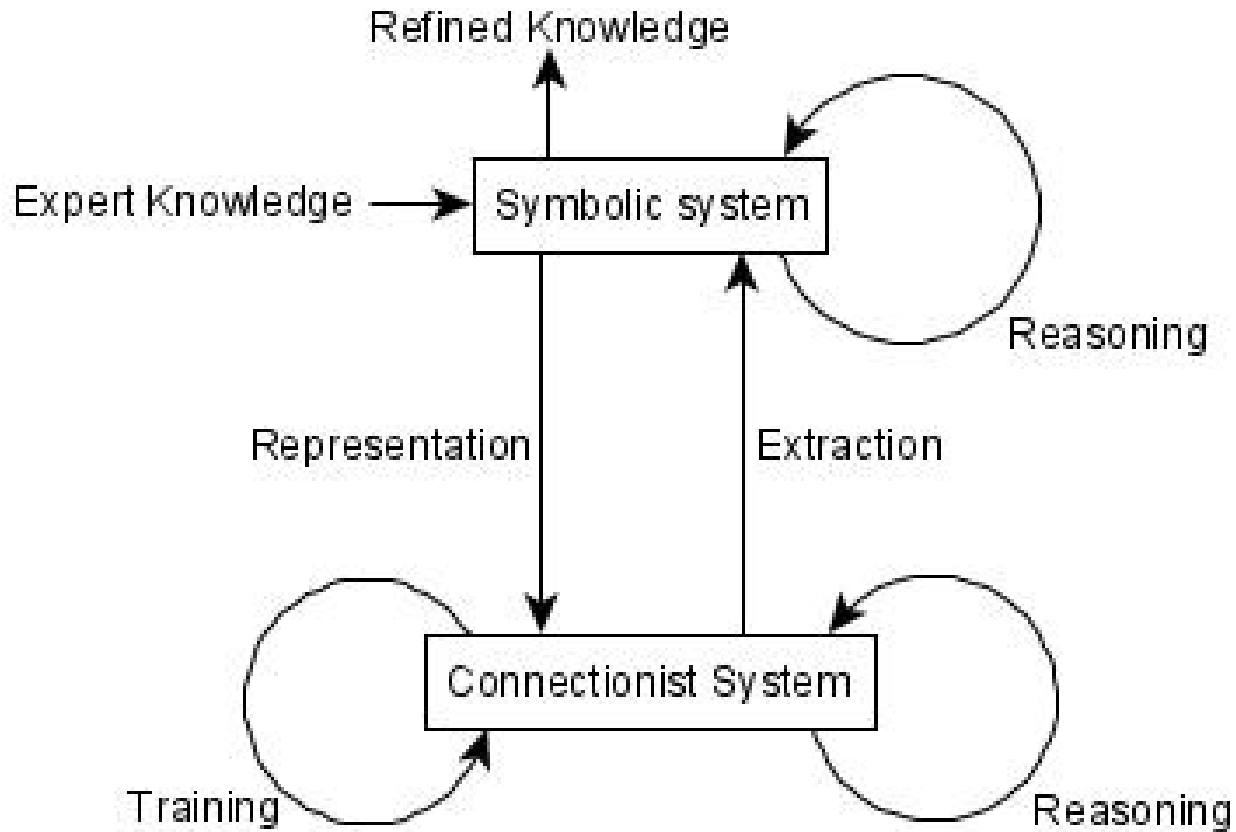


category	number of papers
[symbolic Neuro symbolic]	13
[Symbolic[Neuro]]	9
[Neuro $\cup$ compile(Symbolic)]	10
[Neuro $\rightarrow$ Symbolic]	13
[Neuro[Symbolic]]	0

(6) We finally come to the approach to neuro-symbolic reasoning that I believe has the greatest potential to combine the strengths of logic-based and neural-based AI, namely the **Neuro[Symbolic]** architecture (Figure 15). The basic idea is to embed a symbolic reasoning engine inside a neural engine, with the goal of enabling super-neuro and combinatorial reasoning. The architecture is based on Daniel Kahneman's theory of "thinking fast and



# Neuro-symbolic Learning Cycle



[Bader and Hitzler 2005]



# Some Background

**Workshop Series on Neural-Symbolic Learning and Reasoning, since 2005.**  
Joint with Artur d'Avila Garcez.  
<http://neural-symbolic.org/>

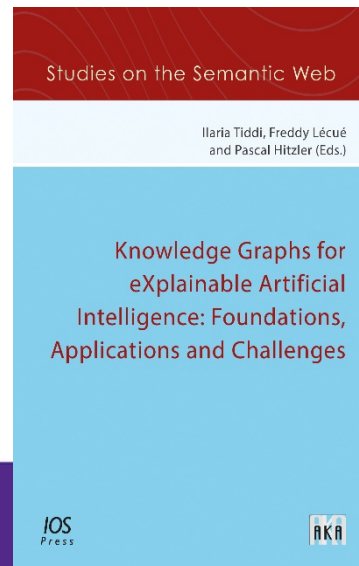
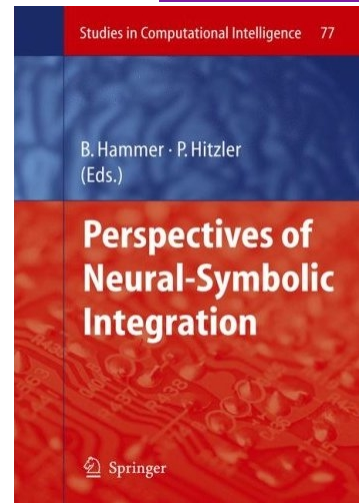
**Barbara Hammer and Pascal Hitzler (eds), Perspectives of Neural-Symbolic Integration, Springer, 2007**

**Neural-Symbolic Learning and Reasoning: A Survey and Interpretation**

**Tarek R. Besold, Artur d'Avila Garcez, Sebastian Bader, Howard Bowman, Pedro Domingos, Pascal Hitzler, Kai-Uwe Kuehnberger, Luis C. Lamb, Daniel Lowd, Priscila Machado Vieira Lima, Leo de Penning, Gadi Pinkas, Hoifung Poon, Gerson Zaverucha**

<https://arxiv.org/abs/1711.03902> (2017)

**Ilaria Tiddi, Freddy Lecue, Pascal Hitzler (eds.), Knowledge Graphs for eXplainable Artificial Intelligence: Foundations, Applications and Challenges. Studies on the Semantic Web Vol. 47, IOS Press, 2020.**





**Thanks!**

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