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Towards a Semantical Hierarchy of Logic Programming Classes

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Logic programs

A (*normal logic*) *program* P is a finite set of *clauses*

$$\forall(A \leftarrow L_1, \ldots, L_n),$$

over a first-order language $\mathcal{L}$, written as

$$A \leftarrow L_1, \ldots, L_n,$$

where A is an atom and the L_i are literals. (A *head*, $L_1, \ldots, L_n$ *body*.)

P is called *definite*, if P does not contain negation symbols.

B_P: Herbrand base (set of all ground instances of atoms).
$I_P = 2^{B_P}$: set of all (2-valued) interpretations (complete lattice wrt. $\subseteq$).
ground(P): set of all ground instances of clauses from P.

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Locally hierarchical/acyclic programs

P *locally hierarchical* (*lh*) (Cavedon 1989)
if exists *level mapping* $l : B_P \to \alpha$ for some ordinal α such that

$$l(A) > l(L_i)$$

for all $A \leftarrow L_1, \ldots, L_n \in \text{ground}(P)$.

P *acyclic* if P lh and $\alpha = \omega \qquad (= \mathbb{N})$.

Immediate consequence operator $T_P : I_P \to I_P$: $T_P(I)$ is set of all $A \in B_P$ for which exists $A \leftarrow \texttt{body} \in \text{ground}(P)$ s.t. $I \models \texttt{body}$.

Fixed points of T_P are exactly the *supported models* of P.

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Acceptable programs I (P^-)

P program, p, q predicate symbols in P.
p *refers to* q iff exists ground clause with p in head and q in body.
p *depends on* q is reflexive transitive closure of "refers to".
Neg_P set of all predicate symbols which occur negatively in P.
Neg_P^* set of all predicate symbols on which the predicate symbols in Neg_P depend.
P^- set of all clauses in ground(P) with head in Neg_P^*.

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Acceptable programs II

P *acceptable* (Apt and Pedreschi 1993) iff
exists level mapping $l : B_P \to \omega$
and model I which is a supported model of P^-
(when restricted to atoms in Neg_P^*)
s.t. for all $A \leftarrow L_1, \dots, L_n \in \mathrm{ground}(P)$
and all $i = 1, \dots, i$ we have

$$I \models L_1 \wedge \cdots \wedge L_{i-1} \qquad \text{implies} \qquad l(A) > l(L_i).$$

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Fitting operators

P program.
For each $A \in B_P$ form *pseudo clause*

$$A \leftarrow \bigvee C_i \quad (= \mathtt{body}_A)$$

where C_i are exactly the bodies of the clauses in ground(P) with head A.

Choose your favourite (suitable) many-valued logic Λ
with space of interpreatations I_Λ
and associate an operator $\Phi_{P,\Lambda} : I_\Lambda \to I_\Lambda$ by

$$\Phi_{P,\Lambda}(I)(A) = I\left(\mathtt{body}_A\right).$$

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3-valued interpretations

3 truth values $\{f, u, t\}$ (false, undefined, true).
Set of all interpretations $I_{P,3}$ is set of pairs (T, F)
with $T, F \subseteq B_P$ and $T \cap F = \emptyset$.
T true atoms, F false atoms, rest undefined

$(T, F) \leq (T', F')$ iff $T \subseteq T'$ and $F \subseteq F'$.

$I_{P,3}$ is complete semilattice.

Fitting operators in the following logics are monotonic, i.e. have least fixed points. With each operator we associate a class of programs determined by the fact that the considered Fitting operator has a least fixed point which leaves nothing undefined.

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Φ-accessible programs I

		C_1	C_2	C_3	D_1	D_2
p	q	$p \wedge q$	$p \wedge q$	$p \wedge q$	$p \vee q$	$p \vee q$
t	t	t	t	t	t	t
t	u	u	u	u	t	u
t	f	f	f	f	t	t
u	t	u	u	u	t	u
u	u	u	u	u	u	u
u	f	f	u	u	u	u
f	t	f	f	f	t	t
f	u	f	f	u	u	u
f	f	f	f	f	f	f

C_1, D_1 associated class: Φ-*accessible programs*.
This is Kleene's strong 3-valued logic
and the original Fitting semantics (Fitting 1985).

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Other classes

C_3, D_2 associated class: locally hierarchical programs
acyclic if least fixed point is reached at $\Phi{\uparrow}\omega$.

C_2, D_2 ssociated class: acceptable programs
if least fixed point if reached at $\Phi{\uparrow}\omega$.

C_1, D_2 associated class: Φ^*-*accessible* programs.
(This class is computationally universal)

P is Φ^*-accessible iff
for each $A \leftarrow L_1, \ldots, L_n$ in ground(P) either
$I \models L_1 \wedge \cdots \wedge L_n$ and $L(A) > l(L_i)$ for all i or
exists i s.t. $I \not\models L_i$, $I \not\models A$ and $l(A) > l(L_i)$.

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Φ-accessible programs II

Alternative characterization

P is Φ-accessible iff
each $A \in B_P$ satisfies either (i) or (ii).

(i) Exists $A \leftarrow L_1, \ldots, L_n$ in ground(P) s.t.
$I \models L_1 \wedge \cdots \wedge L_n$ and $l(A) > l(L_i)$ for all i.

(ii) For each $A \leftarrow L_1, \ldots, L_n$ in ground(P)
exists i with $I \not\models L_i$, $I \not\models A$, $l(A) > l(L_i)$.

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Diagram

$$
\begin{tikzcd}
 & & \text{definite} \arrow[d] \\
\text{acyclic} \arrow[r] \arrow[d] & \text{lh} \arrow[r] \arrow[dd] & \text{loc. strat.} \arrow[ddd] \\
\text{acceptable} \arrow[d] & & \\
{[\Phi^*_\omega]} \arrow[r] \arrow[d] & {[\Phi^*]} \arrow[d] & \\
{[\Phi_\omega]} \arrow[r] & {[\Phi]} \arrow[r] \arrow[d] & \text{weakly strat.-a} \\
 & \text{USM} &
\end{tikzcd}
$$

stratified: Przymusinski 1988
weakly stratified:
Przymusinsy and Przymusinska 1990

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Stable and Supported Models

$I \in I_P$ is *well-supported* if
exists strict well-founded partial order $\prec$ on I
s.t. for any $A \in I$ exists
$A \leftarrow B_1, \ldots, B_n, \neg C_1, \ldots, \neg C_m$ in ground(P)
with $I \models B_1 \wedge B_n \wedge \neg C_1 \wedge \cdots \wedge \neg C_m$
and $B_i \prec A$ for each i.
(Fages 1991, 1994)

Let P be a logic program.
M is well-supported model iff M is stable model.

Let P' be obtained from P by omitting
all negative literals in the clauses.

Let P be program s.t. P' is Φ^*-accessible.
Then M supported model iff M stable model.

$*$ Result does not generalize to $[\Phi]$.