Some corollaries on the fixpoint completion

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Contents

The fixpoint completion of a logic program allows to transform Gelfond-Lifschitz operators (stable semantics) into simpler single-step operators (supported model semantics).

We study some corollaries from this observation.

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Background

[DK89] Phan M. Dung and Kanchana Kanchanasut, A fixpoint approach to declarative semantics of logic programs. In: Ewing L. Lusk and Ross A. Overbeek, Logic Programming, Proceedings of the North American Conference 1989, NACLP'89, Cleveland, Ohio, MIT Press, 1989, pp. 603-625.

Program transformation $P \mapsto \mathrm{fix}(P)$.
Complete unfolding through positive body literals.

[Wen02] Matthias Wendt, Unfolding the well-founded semantics, Journal of Electrical Engineering 53 (12/s), 2002, 56-59.

Shows $\mathrm{GL}_P(I) = T_{\mathrm{fix}(P)}(I)$ for all interpretations I.

The fixpoint completion

Quasi-interpretation Q: set of clauses of form $A \leftarrow \neg B_1, \ldots, \neg B_m$.

Program P: set of (ground) clauses of form

$$A \leftarrow A_1, \ldots, A_n, \neg B_1, \ldots, \neg B_m.$$

$T'_P(Q)$ set of $A \leftarrow \texttt{body}_1, \ldots, \texttt{body}_n, \neg B_1, \ldots, \neg B_m$
where $A \leftarrow A_1, \ldots, A_n, \neg B_1, \ldots, \neg B_m$ in P
and $A_i \leftarrow \texttt{body}_i$ in Q for all i.

$T'_P \uparrow \omega = \mathrm{lfp}(T'_P) = \mathrm{fix}(P)$ quasi-interpretation.

Semantic Operators

$T_P(I)$ set of all A
with $A \leftarrow L_1, \dots, L_n$ in P and $I \models L_1, \dots, L_n$.

P/I set of all $A \leftarrow A_1, \dots, A_n$
with $A \leftarrow A_1, \dots, A_n, \neg B_1, \dots, \neg B_m$ in P
and $I \not\models B_1, \dots, I \not\models B_m$.

$\mathrm{GL}_P(I) = \mathrm{lfp}(T_{P/I})$.

For all interpretations I: $\quad \mathrm{GL}_P(I) = T_{\mathrm{fix}(P)}(I)$. [Wen02]

Iterative Behaviour

P locally hierarchical, i.e. exists level mapping l
with $l(A) > l(L_i)$ for all $A \leftarrow L_1, \ldots, L_n$ and all i.

Then T_P contraction (wrt. some generalized metric).

$T_P^\alpha(I)$ converges to unique supported model of P (for all I).

l maps to $\mathbb{N}$: T_P contraction with respect to metric.

l injective: T_P contraction on Cantor set (via isometry).

[Hitzler and Seda, TCS, to appear]

Iterative Behaviour

P locally stratified, i.e. exists level mapping l
with $l(A) \geq l(A_i)$ and $l(A) > l(B_j)$
for all $A \leftarrow A_1, \ldots, A_n, \neg B_1, \ldots, \neg B_m$ and all i.

Then trivially(!): $\text{fix}(P)$ locally hierarchical.

$\mathrm{GL}_P \equiv T_{\text{fix}(P)}$

GL_P contraction (wrt. some generalized metric).

$\mathrm{GL}_P^\alpha(I)$ converges to unique stable model of P (for all I).

l maps to $\mathbb{N}$: GL_P contraction with respect to metric.

l injective: GL_P contraction on Cantor set (via isometry).

Connectionist Systems

T_P continuous in Cantor topology,
then T_P uniformly approximable by artificial neural network.

P without local variables, then $\mathrm{fix}(P)$ without local variables.
Then $T_{\mathrm{fix}(P)}$ continuous [Seda 1995]. Hence GL_P continuous,
i.e. GL_P approximable by artificial neural network.

etc.

[Hitzler and Seda; Hölldobler, Kalinke, and Störr]

Self-Similarity

An observation by Sebastian Bader.

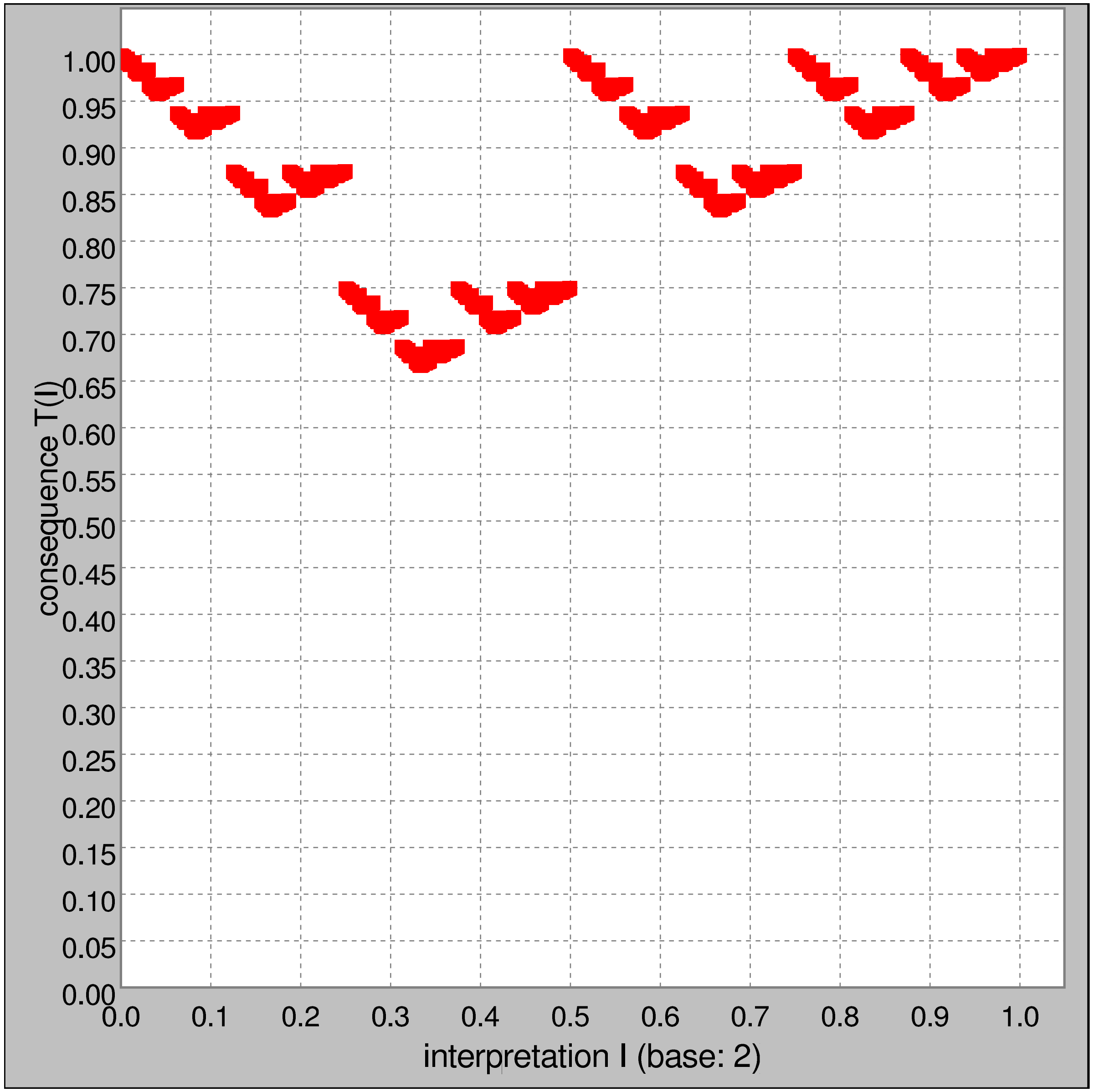
Graph of T_P visualized via embedding into $[0,1] \times [0,1]$ using p-adic numbers.

Graph shows self-similarity.

Is graph a fractal?

(We're working on it.)

The following pictures were produced by Sebastian Bader.

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p(X) :- not p(X).
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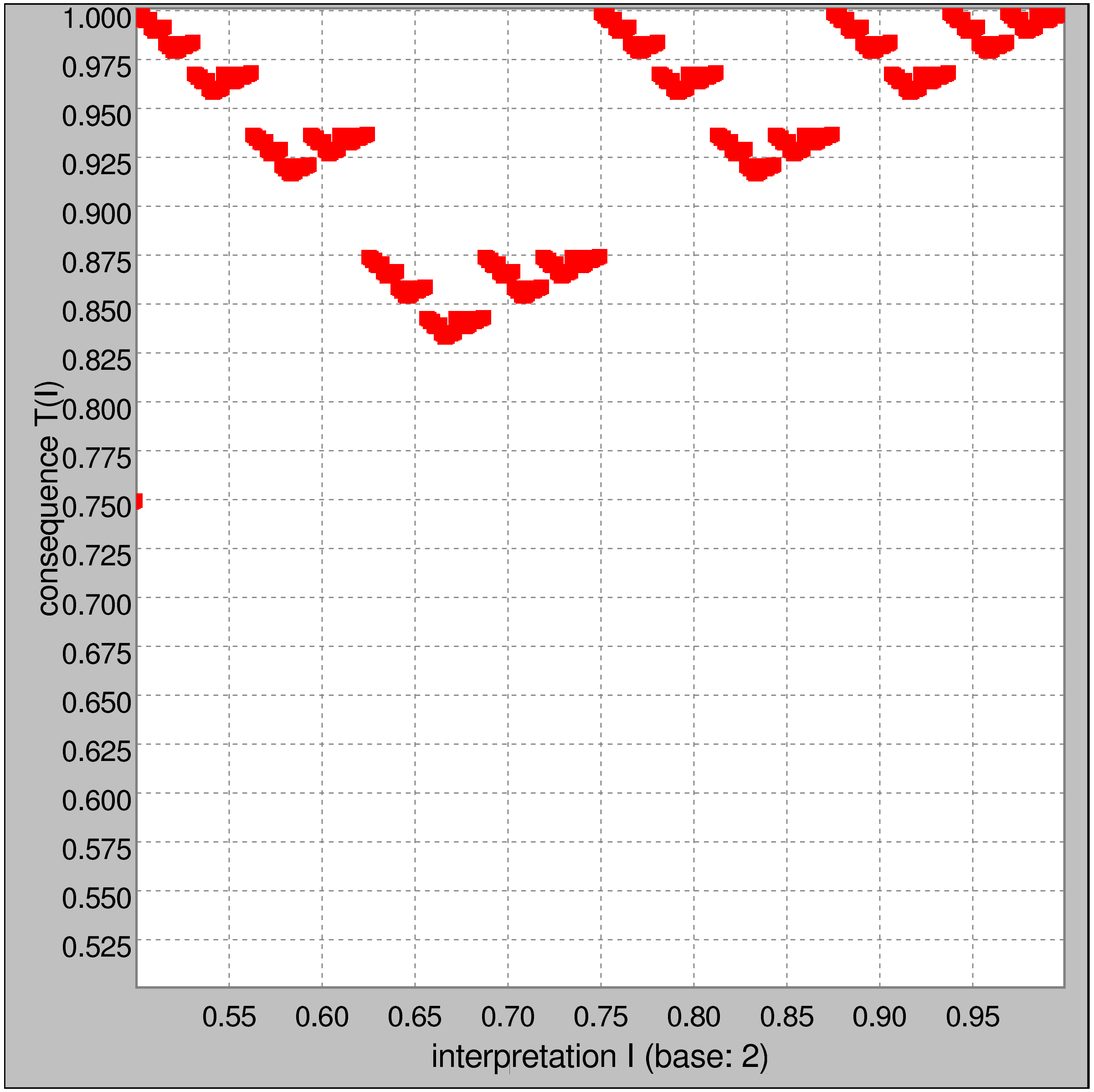


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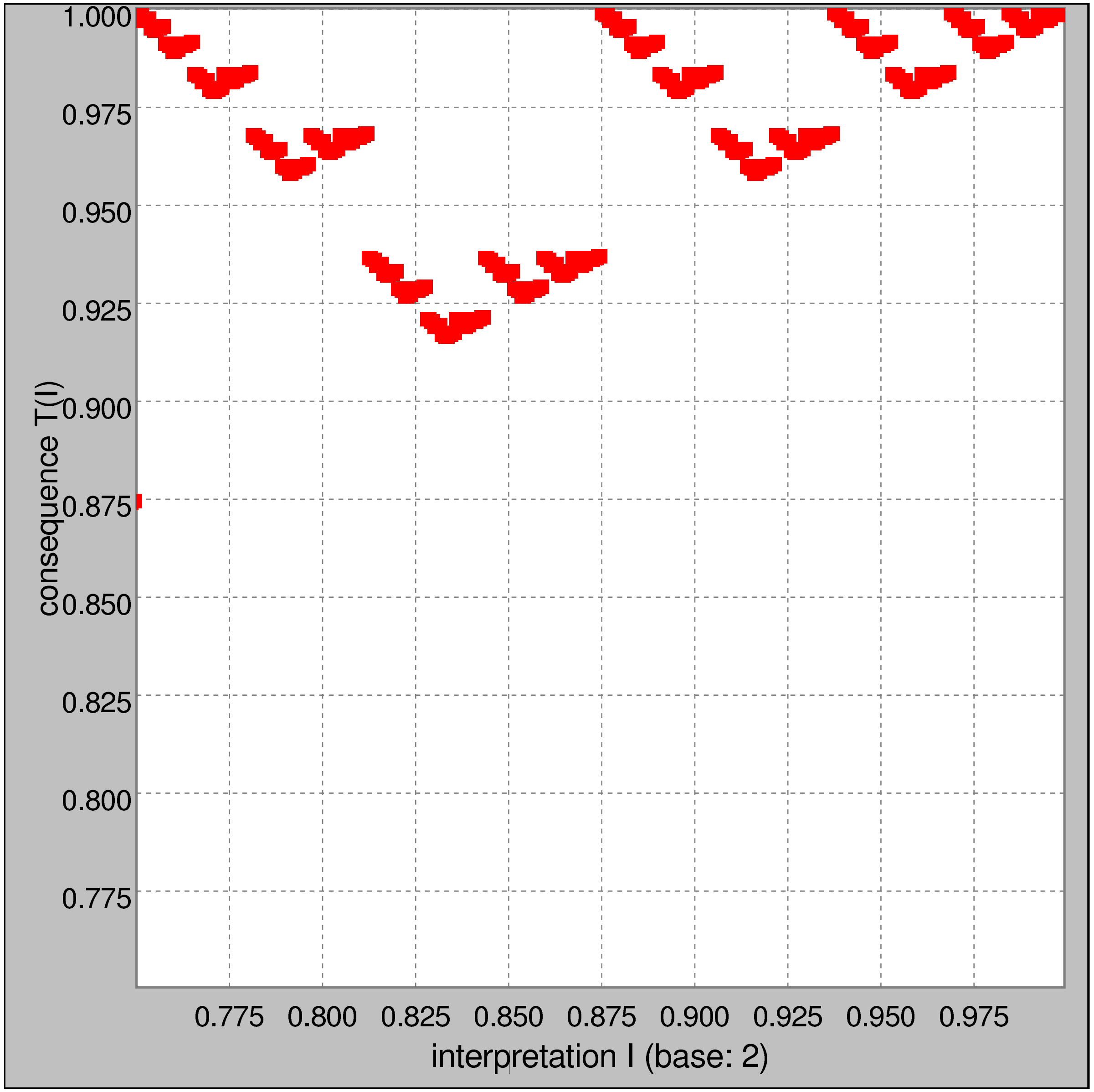


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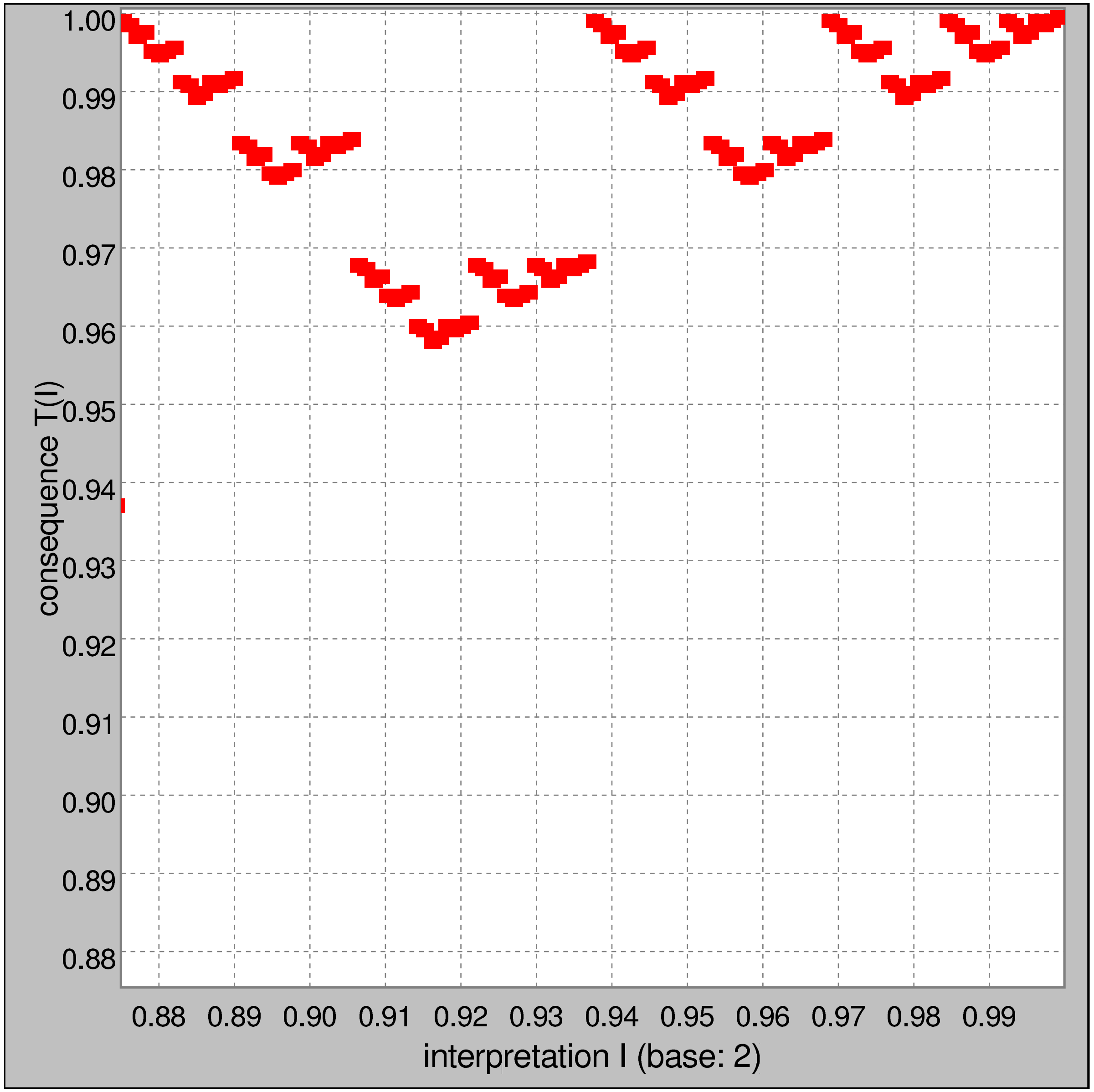


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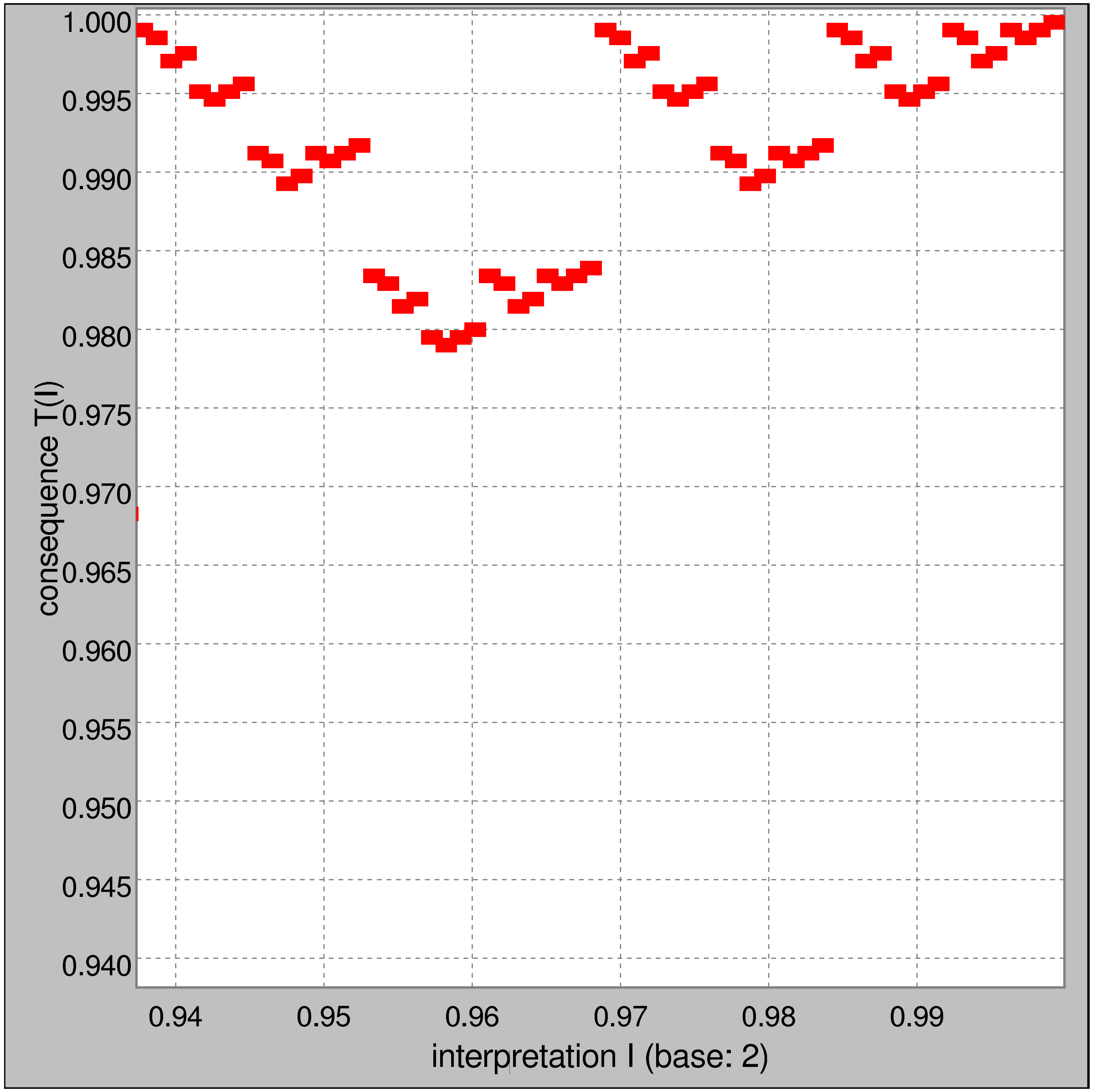


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Self-Similarity

If graph of T_P is self-similar for all P,

then graph of GL_P, too.

By means of Wendt's result.

Conclusions?

All aspects mentioned can be refined.

Almost effortless way of obtaining results on the Gelfond-Lifschitz operator.

Usefulness remains to be determined.