de 1

Circular Belief in Logic Programming Semantics

Pascal Hitzler

KRR colloquium, Dresden, October 2002

Contents

Builds on methodology proposed by Hitzler and Wendt, KI2002 [HW02].

Show how this can be used for developing new and meaningful semantics.

New semantics allows to deal with circular belief.

Artificial Intelligence Institute, Dresden University of Technology, Dresden, Germany

de 2

Notation

We work on ground instantiations of normal logic programs.

Negation symbols may appear in clause bodies.

Essentially, program P is countably infinite set of propositional rules.
Herbrand base $B_P \approx$ set of propositional variables (atoms).

$$A \leftarrow B_1, \dots, B_n, \neg C_1, \dots, \neg C_m$$
$$A \leftarrow \texttt{body}$$

Level mapping $B_P \to \alpha$ for some ordinal α.

Slide 3

Least models

Positive (definite) program P.

There is a unique model M of P for which there exists a level mapping $l : B_P \to \alpha$ such that for each $A \in B_P$ with $M \models A$ there exists $A \leftarrow \texttt{body}$ in P with $M \models \texttt{body}$ and $l(A) > l(B)$ for each $B \in \texttt{body}$.

$M = T_P{\uparrow}\omega = \mathrm{lfp}(T_P)$ is the least model of P.

Slide 4

Stable models

(Fages 1994)

P normal (with negation).

A model M of P is stable if and only if there exists a level mapping $l : B_P \to \alpha$ such that for each $A \in B_P$ with $M \models A$ there exists $A \leftarrow \texttt{body}$ in P with $M \models \texttt{body}$ and $l(A) > l(B)$ for all $B \in \texttt{body}^+$.

$\texttt{body}^+$: all atoms occuring positively in $\texttt{body}$.

$M = \mathrm{GL}_P(M) = T_{P/M}{\uparrow}\omega = \mathrm{lfp}(T_{P/M})$.

de 5

Kleene's strong three-valued logic

Truth values $f < u < t$, $\wedge = \min$, $\vee = \max$, $\neg$ as expected.

Interpretations: consistent signed sets of atoms.
$I = I^{+}\dot{\cup}\neg I^{-} \subseteq B_P \cup \neg B_P$.

I^{+}: true atoms
I^{-}: false atoms

Signed: contains atoms and negated atoms.
Consistent: does not contain both A and $\neg A$.

With order $I \subseteq K$: Plotkin's domain $\mathbb{T}^{\omega}$.

I-partial level mapping:
partial mapping $l : B_P \to \alpha$ with $\mathrm{dom}(l) = I^{+} \cup I^{-}$.
Set $l(\neg A) = l(A)$.

de 6

Fitting models

[HW02]

There is a greatest model M of P such that there is an M-partial level mapping l for P such that each $A \in \mathrm{dom}(l)$ satisfies one of the following conditions.

(Fi) $A \in M$ and there exists $A \leftarrow L_1, \ldots, L_n$ in P such that for all i we have $L_i \in M$ and $l(A) > l(L_i)$.

(Fii) $\neg A \in M$ and for each $A \leftarrow L_1, \ldots, L_n$ in P there exists i with $\neg L_i \in M$ and $l(A) > l(L_i)$.

$M = \Phi_P \uparrow \alpha = \mathrm{lfp}(\Phi_P)$ Fitting model.

Slide 7

Fitting semantics stratified

Replace

(Fii) $\neg A \in M$ and for $A \leftarrow L_1, \ldots, L_n$ in P
there exists i with $\neg L_i \in M$ and $l(A) > l(L_i)$.

by

(Cii) $\neg A \in M$ and for each $A \leftarrow A_1, \ldots, A_n, \neg B_1, \ldots, \neg B_m$ in P one of the following holds:

(Ciia) There exists i with $\neg A_i \in M$ and $l(A) \geq l(A_i)$.

(Ciib) There exists j with $B_j \in M$ and $l(A) > l(B_j)$.

Prevent recursion through negation: Idea behind (local) stratification.
(Apt, Blair & Walker; Przymusinski)

Slide 8

Well-founded models

[HW02]

There is a greatest model M of P such that there is an M-partial level mapping l for P such that each $A \in \mathrm{dom}(l)$ satisfies one of the following conditions.

(Fi) $A \in M$ and there exists $A \leftarrow L_1, \ldots, L_n$ in P such that for all i we have $L_i \in M$ and $l(A) > l(L_i)$.

(Cii) $\neg A \in M$ and for each $A \leftarrow A_1, \ldots, A_n, \neg B_1, \ldots, \neg B_m$ in P one of the following holds:

(Ciia) There exists i with $\neg A_i \in M$ and $l(A) \geq l(A_i)$.

(Ciib) There exists j with $B_j \in M$ and $l(A) > l(B_j)$.

$M = W_P \uparrow \alpha = \mathrm{lfp}(W_P)$ well-founded model.

Well-founded models

Replacing

(Fi) $A \in M$ and there exists $A \leftarrow L_1, \ldots, L_n$ in P such that for all i we have $L_i \in M$ and $l(A) > l(L_i)$.

by

de 9

(Ci) $A \in M$ and there exists $A \leftarrow A_1 \ldots, A_n, \neg B_1, \ldots, \neg B_m$ in P such that $A_i, \neg B_j \in M$, $l(A) \geq l(A_i)$, $l(A) > l(B_j)$.

seems not satisfactory at first sight.

Greatest model may not exist.

Program $p \leftarrow p$, $q \leftarrow \neg p$
has two incomparable models $\{p, \neg q\}$, $\{\neg p, q\}$.

Alternating fixed-point semantics

stable models: $M = \mathrm{GL}_P(M) = T_{P/M}\uparrow\omega$.

de 10

GL_P antitonic, GL_P^2 monotonic.

well-founded model:
$\mathrm{lfp}\left(\mathrm{GL}_P^2\right) \cup \neg\mathrm{gfp}\left(\mathrm{GL}_P^2\right) \quad = \quad \mathrm{lfp}\left(\mathrm{GL}_P^2\right) \cup \neg\mathrm{GL}_P\left(\mathrm{lfp}\left(\mathrm{GL}_P^2\right)\right)$.

Fitting semantics stratified: 2nd version

Well-founded semantics as stratified Fitting semantics asymmetric.

Slide 11

Uses (Fi) and (Cii).

What happens with (Ci) and (Fii)?

Maximally circular stable models

stable models: $M = \mathrm{GL}_P(M) = \mathrm{lfp}(T_{P/M})$.

maxstable models: $M = \mathrm{CGL}_P(M) = \mathrm{gfp}(T_{P/M})$.

Slide 12

CGL_P antitonic, CGL_P^2 monotonic.

Yields variant of alternating fixed-point semantics.

This corresponds to (Ci) and (Fii).

le 13

Maximally circular well-founded model

Corresponds to maximally circular stable model.

There is a greatest model M of P such that there is an M-partial level mapping l for P such that each $A \in \text{dom}(l)$ satisfies one of the following conditions.

(Ci) $A \in M$ and there exists $A \leftarrow A_1 \ldots, A_n, \neg B_1, \ldots, \neg B_m$ in P such that $A_i, \neg B_j \in M$, $l(A) \geq l(A_i)$, $l(A) > l(B_j)$.

(Fii) $\neg A \in M$ and for each $A \leftarrow L_1, \ldots, L_n$ in P there exists i with $\neg L_i \in M$ and $l(A) > l(L_i)$.

le 14

Circular semantics

Also:

Result $\text{gfp}(T_P)$ (definite P) analogous to $\text{lfp}(T_P)$.

Characterization of maxstable models analogous to Fages 1994 for stable models.

Operator CW_P analogous to W_P (for well-founded semantics).

All these fit together.

Slide 15

Greatest models

Positive (definite) program P.

There is a unique supported interpretation M of P for which there exists a level mapping $l : B_P \to \alpha$ such that for each $A \in B_P$ with $M \not\models A$ and for all $A \leftarrow \texttt{body}$ in P there is some $B \in \texttt{body}$ with $M \not\models B$ and $l(A) > l(B)$.

$M = T_P{\downarrow}\omega = \text{gfp}(T_P)$ is the greatest model of P.

Slide 16

Maxstable models

P normal.

A supported interpretation M of P is maxstable if and only if there exists a level mapping $l : B_P \to \alpha$ such that for each $A \in B_P$ with $M \not\models A$ and for all $A \leftarrow A_1, \ldots, A_n, \neg B_1, \ldots, \neg B_m$ in P with $M \not\models B_1, \ldots, B_m$ there is some A_i with $M \not\models A_i$ with $l(A) > l(A_i)$.

Circular semantics

le 17

A circular model of P is a model M of P which is maximal under the condition that P satisfies (Ci) and (Cii) with respect to M and some M-partial level mapping l.

Fitting semantics stratified (symmetric version).

Circular Semantics

le 18

The *circular models* are the maximal models M of P such that there is an M-partial level mapping l for P such that each $A \in \mathrm{dom}(l)$ satisfies one of the following conditions.

(Ci) $A \in M$ and there exists $A \leftarrow A_1 \dots, A_n, \neg B_1, \dots, \neg B_m$ in P such that $A_i, \neg B_j \in M$, $l(A) \geq l(A_i)$, $l(A) > l(B_j)$.

(Cii) $\neg A \in M$ and for each $A \leftarrow A_1, \dots, A_n, \neg B_1, \dots, \neg B_m$ in P one of the following holds:

- (Ciia) There exists i with $\neg A_i \in M$ and $l(A) \geq l(A_i)$.
- (Ciib) There exists j with $B_j \in M$ and $l(A) > l(B_j)$.

Circular belief

Slide 19

$p \leftarrow p$

Two (total) circular models:
$\{\neg p\}$ (the stable model)
$\{p\}$ (the maxstable model)

$p \leftarrow p \quad q \leftarrow q$

Four (total) circular models:
$\{\neg p, \neg q\}$ (the stable model)
$\{p, q\}$ (the maxstable model)
$\{p, \neg q\}$
$\{\neg p, q\}$

Remarkable: Achieved without disjunctions in heads!

Circular belief

Slide 20

$\texttt{penguin}(X) \leftarrow \texttt{penguin}(X), \texttt{bird}(X)$
$\texttt{flies}(X) \leftarrow \texttt{bird}(X), \neg\texttt{penguin}(X)$
$\texttt{bird(bob)} \leftarrow$

Two (total) circular models.

$\{\texttt{bird(bob)}, \texttt{flies(bob)}\}$ (minstable model)
$\{\texttt{bird(bob)}, \texttt{penguin(bob)}\}$ (maxstable model)

le 21

Circular belief

Consider a coffee delivery robot, which has two choices of doors for entering the room where the coffee is to be delivered. It is unknown to the robot whether these doors, or one of them, are open.

$$\texttt{open}(X) \leftarrow \texttt{open}(X), \texttt{door}(X)$$
$$\texttt{deliverable} \leftarrow \texttt{open}(X)$$
$$\texttt{undeliverable} \leftarrow \neg\texttt{deliverable}$$
$$\texttt{door}(1) \leftarrow$$
$$\texttt{door}(2) \leftarrow$$

Four (total) circular models.

$\{\texttt{door}(1), \texttt{door}(2), \texttt{open}(1), \texttt{open}(2), \texttt{deliverable}\}$ (The maxstable model.)

$\{\texttt{door}(1), \texttt{door}(2), \texttt{undeliverable}\}$ (The minstable model.)

$\{\texttt{door}(1), \texttt{door}(2), \texttt{open}(1), \texttt{deliverable}\}$

$\{\texttt{door}(1), \texttt{door}(2), \texttt{open}(2), \texttt{deliverable}\}$

le 22

Three results

P normal program, M a total circular moel. Then $M = T_{P/M}(M)$.

(Converse does not hold.)

P normal program, F the Fitting model, M a circular model. Then $F \subseteq M$.

(Approximation is sharp in general.)

The well-founded and the maximally circular well-founded model are always circular models.

Slide 23

Quo Vadis

- Development of level mapping approach.
  Semantic meta-theory.
  Study extensions like disjunctive/quantitative logic programs.

- Investigate the use of the cirular semantics for Knowledge Representation and Reasoning.

Thanks to Matthias Wendt for valuable feedback.