Paraconsistent Semantics for Hybrid MKNF Knowledge Bases *

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Abstract. Hybrid MKNF knowledge bases, originally based on the stable model semantics, is a mature method of combining rules and Description Logics (DLs). The well-founded semantics for such knowledge bases has been proposed subsequently for better efficiency of reasoning. However, integration of rules and DLs may give rise to inconsistencies, even if they are respectively consistent. Accordingly, reasoning systems based on the previous two semantics will break down. In this paper, we employ the four-valued logic proposed by Belnap, and present a paraconsistent semantics for Hybrid MKNF knowledge bases, which can detect inconsistencies and handle it effectively. Besides, we transform our proposed semantics to the stable model semantics via a linear transformation operator, which indicates that the data complexity in our paradigm is not higher than that of classical reasoning. Moreover, we provide a fixpoint algorithm for computing paraconsistent MKNF models.

1 Introduction

The Semantic Web [3, 11] is a web of data that can be processed directly and indirectly by machines. The essence of the Semantic Web is to describe data on the web by metadata that convey the semantics of the data, and that is expressed by means of ontologies, which are knowledge bases as studied in the field of Knowledge Representation and Reasoning.

The Web Ontology Language OWL [10] has been recommended by the W3C for representing ontologies. However, OWL is not as expressive as needed for modeling some real world problems. For example, it cannot model integrity constraints or closed-world reasoning that may be more suitable in some application scenarios. Consequently, how to improve OWL has been a very important branch of research in the Semantics Web field.

Knowledge representation approaches using rules in the sense of logic programming (LP), which is complementary to modeling in description logics (DLs, which underly OWL, see [11]) with respect to expressivity, have become a mature

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reasoning mechanism in the past thirty years. Thus combining rules and DLs is of continuous interest for the Semantic Web. However, the naive integration of DLs and rules generally leads to undecidable languages.

DLs are monotonic and adhere to the Open World Assumption, while rules are often nonmonotonic and employ the Closed World Assumption. A significant number of different approaches have been proposed for integrating DLs with rules. They can roughly be divided into two kinds: On the one hand, there are homogeneous approaches that unify DLs and LP in a special, unified, knowledge representation language. DLP [8], SWRL [26], nominal schemas [14, 15] and Hybrid MKNF knowledge bases [19] are methods that belong to this kind of approach. On the other hand there are hybrid approaches that view DLs and rules as independent parts, retaining their own reasoning mechanisms. AL-log [4], CARIN [16], HEX-programs [5] and DL+log [22] are all examples of this integration approach.

Among these approaches, Hybrid MKNF knowledge bases, originally based on the stable model semantics [7], is one of the most mature integration methods. It has favourable properties of decidability, flexibility, faithfulness and tightness. A well-founded semantics [25] for such knowledge bases has been proposed subsequently for better efficiency of reasoning [12, 13]. However, an integration of a rules knowledge base and a DL knowledge base may lead to inconsistencies, even if both of the knowledbe bases are consistent if taken alone. Accordingly, reasoning systems based on the previous two semantics will break down. Therefore it is necessary to present a new semantics of hybrid MKNF knowledge bases for handling inconsistencies.

Traditionally there are two kinds of approaches to handle inconsistencies, one of which is by repairing the knowledge base and threby recovering consistency [24, 9]. But this approach may cause some new problems, such as different results caused by different methods of recovering consistency, inability of reusing some information that is eliminated, and so on. The other method admits inconsistencies and deals with them directly in a paraconsistent logic, and usually a four-valued logic [2, 21, 23, 18] is chosen for this purpose. Due to the limitations of the first method, we adopt the second one in the sequel.

The remainder of the paper is as follows. In section 2, we recall preliminaries on the Description Logic $\mathcal{ALC}$ and on Hybrid MKNF Knowledge Bases. In section 3, we propose our paraconsistent semantics for Hybrid MKNF knowledge bases, and obtain some properties of it. In section 4, we present a transformation from paraconsistent semantics to the stable model semantics of hybrid MKNF knowledge bases. In section 5, we characterize the paraconsistent MKNF models via a fixpoint operator. We conclude and discuss future work in section 6.

Due to space limitations, proofs are omitted in this paper. They are available from http://www.pascal-hitzler.de/resources/publications/para-hmknf-tr.pdf.

2 Preliminaries

In this section, we introduce notions and notation used in the sequel.

Syntax	Name	Semantics
A	Atomic concept	$A^I \subseteq \triangle^I$
R	Atomic role	$R^I \subseteq \triangle^I \times \triangle^I$
o	Individual	$o^I \in \triangle^I$
$\bot$	Bottom concept	$\bot^I = \emptyset$
$\top$	Top concept	$\top^I = \triangle^I$
$\neg C$	Concept negation	$\triangle^I \setminus C^I$
$C \sqcap D$	Concept intersection	$C^I \cap D^I$
$C \sqcup D$	Concept union	$C^I \cup D^I$
$\exists R.C$	Existential quantifier	$\{x \mid \exists y \in \triangle^I : (x,y) \in R^I \wedge y \in C^I\}$
$\forall R.C$	Universal quantifier	$\{x \mid \forall y \in \triangle^I : (x,y) \in R^I \rightarrow y \in C^I\}$
$C \sqsubseteq D$	Inclusion axiom	$C^I \subseteq D^I$
$C(a)$	Assertion axiom	$a^I \in C^I$
$R(a,b)$	Assertion axiom	$(a^I, b^I) \in R^I$

Table 1. Syntax and Semantics of $\mathcal{ALC}$

2.1 The Description Logics $\mathcal{ALC}$

We briefly recall the description language $\mathcal{ALC}$, which is the logical basis of OWL. For further background about description logic, please refer to [1].

Elementary descriptions are atomic concepts, atomic roles and individuals. Complex concepts are constructed via connectors in $\mathcal{ALC}$ inductively by the rules as presented in table 1. An $\mathcal{ALC}$ knowledge base $\mathcal{O}$ consists of a TBox $\mathcal{T}$, which is a finite set of inclusion axioms, and an ABox $\mathcal{A}$, which is a finite set of assertion axioms. Inclusion axioms and assertion axioms have the form presented in table 1.

The semantics of $\mathcal{ALC}$ knowledge bases $\mathcal{O}$ is defined by means of interpretations $I = (\cdot^I, \triangle^I)$, where $\triangle^I$ is a nonempty set (i.e., the domain of the interpretation) and $\cdot^I$ is a function that assigns a set $A^I \subseteq \triangle^I$ to each concept A and a binary relation $R^I \subseteq \triangle^I \times \triangle^I$ to each role R as in table 1. An interpretation I is a *model* of $\mathcal{O}$ if it satisfies all the axioms in $\mathcal{T}$ and $\mathcal{A}$. A concept A is *satisfiable* with respect to $\mathcal{O}$ if there exists a model I of $\mathcal{O}$ such that $A^I \neq \emptyset$. Furthermore, $\mathcal{O}$ *entails* an axiom α, written $\mathcal{O} \models \alpha$, if α is true in all models of $\mathcal{O}$.

2.2 Four-valued logic

Four-valued logics have been studied mainly in the propositional case. The basic idea is to substitute four truth values for the two truth values used in classical logic: the four truth values are $\mathbf{t}, \mathbf{f}, \bot$ and $\top$, representing *true, false, contradictory* (both *true* and *false*) and *unknown* (neither *true* nor *false*) respectively. Moreover, with two partial orders $\leq_k$ and $\leq_t$, that stand for a measure of the amount of information and a measure of truth, respectively, the set consisting of the four truth values becomes the bilattice $\mathcal{FOUR}$ [2] as shown in Fig. 1. In our semantics presented in section 3, we will adopt the partial order $\leq_k$.

Syntactically, four-valued logic is similar to classical logic. The only difference is that there are three types of implications in four-valued logic: inclusion implication $\supset$, material implication $\to$ and strong implication $\leftrightarrow$ defined as in [2]. In our approach, we will employ inclusion implication that will be presented later in section 3.

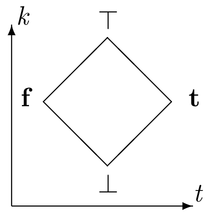

Fig. 1. $\mathcal{FOUR}$

Semantically, a *paraconsistent interpretation* of a four-valued logic knowledge base is defined as a function $\mathcal{I}$, mapping each proposition L to a truth value:

$$L^{\mathcal{I}} = \begin{cases} \mathbf{t} & \text{iff } L \in \mathcal{I} \text{ and } \neg L \notin \mathcal{I} \\ \mathbf{f} & \text{iff } L \notin \mathcal{I} \text{ and } \neg L \in \mathcal{I} \\ \top & \text{iff } L \in \mathcal{I} \text{ and } \neg L \in \mathcal{I} \\ \bot & \text{iff } L \notin \mathcal{I} \text{ and } \neg L \notin \mathcal{I} \end{cases}$$

As for other formulae in four-valued logic, $\mathcal{I}$ evaluates them inductively as follows: (i) $(L \wedge R)^{\mathcal{I}} = L^{\mathcal{I}} \wedge R^{\mathcal{I}}$; (ii) $(L \vee R)^{\mathcal{I}} = L^{\mathcal{I}} \vee R^{\mathcal{I}}$; (iii) $(\neg L)^{\mathcal{I}} = \neg L^{\mathcal{I}}$, where L and R are formulae in Φ, $\wedge$ and $\vee$ are meet and join in $\mathcal{FOUR}$.

The designated truth value set in four-valued logic is $\{\mathbf{t}, \top\}$. A paraconsistent interpretation $\mathcal{I}$ is a *paraconsistent model* of a set of formulas Φ, iff it evaluates every formula in Φ to $\mathbf{t}$ or $\top$. Φ *paraconsistently entails* a formula L, written $\Phi \models_4 L$, iff every paraconsistent model of Φ is a paraconsistent model of L.

2.3 The Logic of Minimal Knowledge and Negation as Failure

The Logic of Minimal Knowledge and Negation as Failure (MKNF) [17] has been proposed as unifying framework for different nonmonotonic formalisms, such as default logic, autoepistemic logic, and logic programming [19].

Let Σ be the signature that consists of first-order predicates, constants and function symbols, plus the binary equality predicate $\approx$. A first-order atom $P(t_1, \ldots, t_l)$ is an MKNF formula, where P is a first-order predicate and t_i are first-order terms. Other MKNF formulae are built over Σ using standard connectives in first-order logic and two extra modal operators, **K** and **not**, as follows: $\neg\varphi$, $\varphi_1 \wedge \varphi_2$, $\exists x : \varphi$, $\mathbf{K}\varphi$, $\mathbf{not}\varphi$. Formulae of the form $\mathbf{K}\varphi$ ($\mathbf{not}\varphi$) are called *modal* **K**-*atoms* (**not**-*atoms*). An MKNF formula φ is *ground* if it contains no variables, and *closed* if it has no free variables in it. $\varphi[t/x]$ is the formula obtained from φ by substituting the term t for the variable x.

Apart from the constants occurring in the formulae, we assume that there is an infinite supply of constants. The set of all these constants constitutes the Herbrand universe of the formulae, denoted by $\triangle$. Let I be the Herbrand first-order interpretation over Σ and $\triangle$, φ a closed MKNF formula, then *satisfiability* of φ is defined inductively as

$(I, M, N) \models P(t_1, \ldots, t_l)$ iff $P(t_1, \ldots, t_l) \in I$
$(I, M, N) \models \neg\varphi$ iff $(I, M, N) \not\models \varphi$
$(I, M, N) \models \varphi_1 \wedge \varphi_2$ iff $(I, M, N) \models \varphi_1$ and $(I, M, N) \models \varphi_2$

$(I, M, N) \models \exists x : \varphi$ iff $(I, M, N) \models \varphi[\alpha/x]$ for some $\alpha \in \triangle$
$(I, M, N) \models \mathbf{K}\varphi$ iff $(J, M, N) \models \varphi$ for all $J \in M$
$(I, M, N) \models \mathbf{not}\varphi$ iff $(J, M, N) \not\models \varphi$ for some $J \in N$
where M and N are nonempty sets of Herbrand first-order interpretations. M is an MKNF *model* of φ if: (i) $(I, M, M) \models \varphi$ for each $I \in M$; (ii) for each set of Herbrand first-order interpretations M' such that $M' \supset M$, we have $(I', M', M) \not\models \varphi$ for some $I' \in M'$. $\varphi \models_{MKNF} \psi$ if and only if $(I, M, M) \models \psi$, for all the models M of φ, and all $I \in M$.

2.4 Hybrid MKNF Knowledge Bases

Hybrid MKNF knowledge bases, based on MKNF, is an approach for integrating Description Logics and Logic Programming proposed by Boris Motik and Riccardo Rosati [19]. It consists of a finite number of MKNF rules and a decidable description logic knowledge base which can be translated to first-order logic equivalently.

Definition 1 *Let $\mathcal{O}$ be a DL knowledge base. A first-order function-free atom $P(t_1, \ldots, t_n)$ over Σ such that P is $\approx$ or it occurs in $\mathcal{O}$ is called a* DL-atom*; all other atoms are called* non-DL-atoms*. An* MKNF *rule r has the following form where H_i, A_i, B_i are first-order function-free atoms:*

$$\mathbf{K}H_1 \vee \ldots \vee \mathbf{K}H_n \leftarrow \mathbf{K}A_{n+1} \wedge \ldots \wedge \mathbf{K}A_m \wedge \mathbf{not}B_{m+1} \wedge \ldots \wedge \mathbf{not}B_k \quad (1)$$

The sets $\{\mathbf{K}H_i\}$, $\{\mathbf{K}A_i\}$, $\{\mathbf{not}B_i\}$ are called the rule head, the positive body and the negative body, respectively. An MKNF rule r is nondisjunctive if $n = 1$; r is positive if $m = k$; r is a fact if $m = k = 0$. A program $\mathcal{P}$ is a finite set of MKNF rules. A hybrid MKNF knowledge base $\mathcal{K}$ is a pair ($\mathcal{O}$,$\mathcal{P}$).

By translating MKNF rules and description logic expressions to MKNF formulae, the semantics of $\mathcal{K}$ is obtained.

Definition 2 *Let $\mathcal{K}$ = ($\mathcal{O}$,$\mathcal{P}$) be a hybrid knowledge base. We extend π to r, $\mathcal{P}$, and $\mathcal{K}$ as follows, where x is the vector of the free variables of r:*

$$\pi(r) = \forall x : (\mathbf{K}H_1 \vee \ldots \vee \mathbf{K}H_n \subset \mathbf{K}A_{n+1} \wedge \ldots \wedge \mathbf{K}A_m \wedge \mathbf{not}B_{m+1} \wedge \ldots \wedge \mathbf{not}B_k)$$

$$\pi(\mathcal{P}) = \wedge_{r \in \mathcal{P}} \pi(r) \qquad \pi(\mathcal{K}) = \mathbf{K}\pi(\mathcal{O}) \wedge \pi(\mathcal{P})$$

Note that $\pi(\mathcal{O})$ consists of first-order formulae translated from $\mathcal{O}$ in the way as defined in [20].

$\mathcal{K}$ is satisfiable if and only if an MKNF model of $\pi(\mathcal{K})$ exists, and $\mathcal{K}$ entails a closed MKNF formula φ, written $\mathcal{K} \models \varphi$ if and only if $\pi(\mathcal{K}) \models_{MKNF} \varphi$.

To ensure that the MKNF logic is decidable, *DL-safety* is introduced as a restriction to MKNF rules.

Definition 3 *An MKNF rule is DL-safe if every variable in r occurs in at least one non-DL-atom* **K**B *occurring in the body of r. A hybrid MKNF knowledge base $\mathcal{K}$ is DL-safe if all its rules are DL-safe.*

In the rest of this paper, without explicitly stating it, hybrid MKNF knowledge bases are considered to be *DL-safe*.

Definition 4 *Given a hybrid MKNF knowledge base $\mathcal{K}$ = ($\mathcal{O}$,$\mathcal{P}$). The ground instantiation of $\mathcal{K}$ is the knowledge base $\mathcal{K}_G$ = ($\mathcal{O}$,$\mathcal{P}_G$), where $\mathcal{P}_G$ is obtained from $\mathcal{P}$ by replacing each rule r of $\mathcal{P}$ with a set of rules substituting each variable in r with constants from $\mathcal{K}$ in all possible ways.*

Grounding the knowledge base $\mathcal{K}$ ensures that rules in $\mathcal{P}$ apply only to objects that occur in $\mathcal{K}$.

Proposition 1 *Let $\mathcal{K}_G$ = ($\mathcal{O}$,$\mathcal{P}_G$) be the grounding instantiation of $\mathcal{K}$ = ($\mathcal{O}$,$\mathcal{P}$). Then the MKNF models of $\mathcal{K}_G$ and $\mathcal{K}$ coincide.*

3 Paraconsistent Semantics for Hybrid MKNF Knowledge Base

Inconsistencies may arise when DLs are integrated with rules. And classical reasoners will break down when they encounter contradictory information. Thus it is necessary to propose a new semantics for the Hybrid MKNF Knowledge Base to handle inconsistencies. In this section, we use four-valued logic as the logical basis when defining the paraconsistent semantics of hybrid MKNF knowledge bases.

In hybrid MKNF knowledge bases with our paraconsistent semantics, syntax differ with the original one slightly.

Definition 5 *Let $\mathcal{O}$ be a DL knowledge base. A first-order function-free atom $P(t_1, \ldots, t_n)$ over Σ such that P is $\approx$ or it occurs in $\mathcal{O}$ is called a* DL-atom*; all other atoms are called* non-DL-atoms*. L is a literal if it is an atom P, or of the form $\neg P$, where P is an atom. An* MKNF *rule r has the following form where H_i, A_i, B_i are first-order function-free literals:*

$$\mathbf{K}H_1 \vee, \ldots, \vee\mathbf{K}H_n \leftarrow \mathbf{K}A_{n+1} \wedge, \ldots, \wedge\mathbf{K}A_m, \mathbf{not}B_{m+1} \wedge, \ldots, \wedge\mathbf{not}B_k \quad (2)$$

The sets $\{\mathbf{K}H_i\}$, $\{\mathbf{K}A_i\}$, $\{\mathbf{not}B_i\}$ are called the rule head, the positive body and the negative body, respectively. An MKNF rule r is nondisjunctive if $n = 1$; r is positive if $m = k$; r is a fact if $m = k = 0$. A program $\mathcal{P}$ is a finite set of MKNF rules. A hybrid MKNF knowledge base $\mathcal{K}$ is a pair ($\mathcal{O}$,$\mathcal{P}$).

Note that we substitute literals for atoms in MKNF rules. In our paradigm, negative literals have the same status as positive literals, and we consider a modified version of Herbrand first-order interpretations, namely the set of ground literals occurring in $\mathcal{K}$, and call them *paraconsistent Herbrand first-order interpretations*. In the rest of the paper, unless explicitly stated, we consider only literals when referring to MKNF rules. Furthermore, the DL-part in a hybrid MKNF knowledge base $\mathcal{K} = (\mathcal{O},\mathcal{P})$ is taken to be $\mathcal{ALC}$ for simplicity.

Definition 6 *A four-valued (paraconsistent) MKNF structure $(\mathcal{I}, \mathcal{M}, \mathcal{N})$ consists of a paraconsistent Herbrand first-order interpretation $\mathcal{I}$ and two nonempty sets of paraconsistent Herbrand first-order interpretations $\mathcal{M}$ and $\mathcal{N}$. $\mathcal{M}$ is called a paraconsistent MKNF interpretation.*

$\mathcal{I}$ is supposed to interpret first-order formulae, while $\mathcal{M}$ and $\mathcal{N}$ are used to evaluate modal **K**-atoms and modal **not**-atoms. MKNF formulae are assigned to the lattice $\mathcal{FOUR}$.

Definition 7 *Let $(\mathcal{I}, \mathcal{M}, \mathcal{N})$ be a paraconsistent MKNF structure, and $\{\mathbf{t}, \mathbf{f}, \bot, \top\}$ be the set of truth values. We evaluate MKNF formulae inductively as follows:*

$$(\mathcal{I}, \mathcal{M}, \mathcal{N})(P(t_1, \dots, t_l)) = \begin{cases} \mathbf{t} & \textit{iff } P(t_1, \dots, t_l) \in \mathcal{I} \textit{ and } \neg P(t_1, \dots, t_l) \notin \mathcal{I} \\ \mathbf{f} & \textit{iff } P(t_1, \dots, t_l) \notin \mathcal{I} \textit{ and } \neg P(t_1, \dots, t_l) \in \mathcal{I} \\ \top & \textit{iff } P(t_1, \dots, t_l) \in \mathcal{I} \textit{ and } \neg P(t_1, \dots, t_l) \in \mathcal{I} \\ \bot & \textit{iff } P(t_1, \dots, t_l) \notin \mathcal{I} \textit{ and } \neg P(t_1, \dots, t_l) \notin \mathcal{I} \end{cases}$$

$$(\mathcal{I}, \mathcal{M}, \mathcal{N})(\neg\varphi) = \begin{cases} \mathbf{t} & \textit{iff } (\mathcal{I}, \mathcal{M}, \mathcal{N})(\varphi) = \mathbf{f} \\ \mathbf{f} & \textit{iff } (\mathcal{I}, \mathcal{M}, \mathcal{N})(\varphi) = \mathbf{t} \\ \top & \textit{iff } (\mathcal{I}, \mathcal{M}, \mathcal{N})(\varphi) = \top \\ \bot & \textit{iff } (\mathcal{I}, \mathcal{M}, \mathcal{N})(\varphi) = \bot \end{cases}$$

$$(\mathcal{I}, \mathcal{M}, \mathcal{N})(\varphi_1 \wedge \varphi_2) = (\mathcal{I}, \mathcal{M}, \mathcal{N})(\varphi_1) \wedge (\mathcal{I}, \mathcal{M}, \mathcal{N})(\varphi_1)$$

$$(\mathcal{I}, \mathcal{M}, \mathcal{N})(\exists x : \varphi) = \bigvee_{\alpha \in \triangle} (\mathcal{I}, \mathcal{M}, \mathcal{N})(\varphi[\alpha/x])$$

$$(\mathcal{I}, \mathcal{M}, \mathcal{N})(\mathbf{K}\varphi) = \bigwedge_{\mathcal{J} \in \mathcal{M}} (\mathcal{J}, \mathcal{M}, \mathcal{N})(\varphi)$$

$$(\mathcal{I}, \mathcal{M}, \mathcal{N})(\varphi_1 \supset \varphi_2) = \begin{cases} \mathbf{t} & \textit{iff } (\mathcal{I}, \mathcal{M}, \mathcal{N})(\varphi_1) \in \{\mathbf{f}, \bot\} \\ (\mathcal{I}, \mathcal{M}, \mathcal{N})(\varphi_2) & \textit{otherwise} \end{cases}$$

$$(\mathcal{I}, \mathcal{M}, \mathcal{N})(\mathbf{not}\varphi) = \begin{cases} \mathbf{t} & \textit{iff } (\mathcal{J}, \mathcal{M}, \mathcal{N})(\varphi) = \bot \textit{ for some } \mathcal{J} \in \mathcal{N} \\ \mathbf{f} & \textit{iff } (\mathcal{J}, \mathcal{M}, \mathcal{N})(\varphi) = \mathbf{t} \textit{ for all } \mathcal{J} \in \mathcal{N} \\ \top & \textit{iff } \exists\, \mathcal{J} \in \mathcal{N} \textit{ s. t. } (\mathcal{J}, \mathcal{M}, \mathcal{N})(\varphi) = \mathbf{f} \\ & \textit{and no other } \mathcal{J} \in \mathcal{N},\ \textit{s. t. } (\mathcal{J}, \mathcal{M}, \mathcal{N})(\varphi) = \bot \\ \bot & \textit{iff } (\mathcal{J}, \mathcal{M}, \mathcal{N})(\varphi) = \top \textit{ for all } \mathcal{J} \in \mathcal{N} \end{cases}$$

Definition 8 *Let $(\mathcal{I}, \mathcal{M}, \mathcal{N})$ be a paraconsistent MKNF structure, the paraconsistent satisfaction of a closed MKNF formula is defined inductively as follows.*
$(\mathcal{I}, \mathcal{M}, \mathcal{N}) \models_4 P(t_1, \ldots, t_l)$ *iff* $P^{\mathcal{I}}(t_1, \ldots, t_l) \in \{\mathbf{t}, \top\}$
$(\mathcal{I}, \mathcal{M}, \mathcal{N}) \models_4 \neg\varphi$ *iff* $(\mathcal{I}, \mathcal{M}, \mathcal{N})\varphi \in \{\mathbf{f}, \top\}$
$(\mathcal{I}, \mathcal{M}, \mathcal{N}) \models_4 \varphi_1 \wedge \varphi_2$ *iff* $(\mathcal{I}, \mathcal{M}, \mathcal{N}) \models_4 \varphi_i$, $i = 1, 2$
$(\mathcal{I}, \mathcal{M}, \mathcal{N}) \models_4 \exists x : \varphi$ *iff* $(\mathcal{I}, \mathcal{M}, \mathcal{N}) \models_4 \varphi[\alpha/x]$ *for some* $\alpha \in \triangle$
$(\mathcal{I}, \mathcal{M}, \mathcal{N}) \models_4 \varphi_1 \supset \varphi_2$ *iff* $(\mathcal{I}, \mathcal{M}, \mathcal{N}) \not\models_4 \varphi_1$ *or* $(\mathcal{I}, \mathcal{M}, \mathcal{N}) \models_4 \varphi_2$
$(\mathcal{I}, \mathcal{M}, \mathcal{N}) \models_4 \mathbf{K}\varphi$ *iff* $(\mathcal{J}, \mathcal{M}, \mathcal{N}) \models_4 \varphi$ *for all* $\mathcal{J} \in \mathcal{M}$
$(\mathcal{I}, \mathcal{M}, \mathcal{N}) \models_4 \mathbf{not}\varphi$ *iff* $(\mathcal{J}, \mathcal{M}, \mathcal{N}) \not\models_4 \varphi$ *for some* $\mathcal{J} \in \mathcal{N}$

It can be easily verified that Definition 7 of paraconsistent semantics is compatible with Definition 8 of paraconsistent satisfaction. Now we define paraconsistent MKNF models of MKNF formulae and hybrid MKNF KBs by paraconsistent satisfaction.

Definition 9 *A paraconsistent MKNF interpretation $\mathcal{M}$ is a paraconsistent MKNF model of a given closed MKNF formula φ, written $\mathcal{M} \models^4_{MKNF} \varphi$ if and only if the following two conditions are satisfied :*

(1) $(\mathcal{I}, \mathcal{M}, \mathcal{M}) \models_4 \varphi$;
(2) for each interpretation $\mathcal{M}'$, $\mathcal{M}' \supset \mathcal{M}$, there exists an $\mathcal{I}' \in \mathcal{M}'$ such that $(\mathcal{I}', \mathcal{M}', \mathcal{M}) \not\models_4 \varphi$.

For a hybrid MKNF KB $\mathcal{K} = (\mathcal{O}, \mathcal{P})$, $\mathcal{K}$ is satisfiable iff a paraconsistent MKNF model of $\pi(\mathcal{K})$ exists. $\varphi \models^4_{MKNF} \phi$, iff $\mathcal{M} \models_4 \phi$ for each paraconsistent MKNF model $\mathcal{M}$ of φ.

Paraconsistent semantics in our paradigm is faithful. That is to say, the semantics yields the paraconsistent semantics for DLs according to [18] when no rules are present, and the p-stable model of LP from [23] when the DL-component is empty.

In order to show this conclusion, we first recall the notion of p-stable model of a program.

Definition 10 ([23]) *Let P be an extended disjunctive program and I a subset of $\mathcal{L}_P$. The reduct of P w.r.t. I is the positive extended disjunctive program P^I such that a clause*

$$L_1 \vee \ldots \vee L_l \leftarrow L_{l+1} \wedge \ldots \wedge L_m$$

is in P^I iff there is a ground clause of the form

$$L_1 \vee \ldots \vee L_l \leftarrow L_{l+1} \wedge \ldots \wedge L_m \wedge notL_{m+1} \wedge \ldots \wedge notL_n \tag{3}$$

from P such that $\{L_{m+1}, \ldots, L_n\} \cap I = \emptyset$. Then I is called a paraconsistent stable model (shortly, p-stable model) of P if I is a p-minimal model of P^I. For a positive extended disjunctive program P, an interpretation I is a model of P if I satisfies every ground clause from P, and a p-minimal model if there exists no model J of P such that $J \subset I$.

Proposition 2 *Let $\mathcal{K}$ = ($\mathcal{O}$,$\mathcal{P}$) be a hybrid MKNF knowledge base, φ a closed first-order formula, and A a ground literal.*

- ⋄ *If $\mathcal{P} = \emptyset$, then $\mathcal{K} \models^4_{MKNF} \varphi$ iff $\mathcal{O} \models_4 \varphi$, where we mean $\mathcal{O} \models_4 \varphi$ as the definition in [18], that is to say, every 4-model of $\mathcal{O}$ is a 4-model of φ.*
- ⋄ *If $\mathcal{O} = \emptyset$, then $\mathcal{K} \models^4_{MKNF} A$ iff $P \models_4 A$, where $P \models_4 A$ means for all the p-stable models I of P, $I \models_4 A$.*

Proof. Case 1: $\mathcal{P} = \emptyset$.(Necessity) If $\mathcal{P} = \emptyset$, then $\mathcal{K}$ consists only of DL axioms. Thus $\mathcal{K} \models^4_{MKNF} \varphi$, equals to $\mathbf{K}\pi(\mathcal{O}) \models^4_{MKNF} \varphi$, which means $\mathcal{M} \models_4 \varphi$ for each paraconsistent MKNF model $\mathcal{M}$ of $\mathbf{K}\pi(\mathcal{O})$. Therefore, $\mathcal{I} \in \mathcal{M}$ iff $\mathcal{I}$ is the paraconsistent model of $\pi(\mathcal{O})$. From $\mathcal{M} \models_4 \varphi$ we can infer $\pi(\mathcal{O}) \models_4 \varphi$, which is the first-order form of $\mathcal{O} \models \varphi$.
(Sufficiency) Similarly, the sufficiency can easily be proved by contradiction.

Case 2: $\mathcal{O} = \emptyset$. If $\mathcal{O} = \emptyset$, then $\mathcal{K}$ consists only of MKNF rules. $\mathcal{K} \models^4_{MKNF} \varphi$ if and only if $\pi(\mathcal{P}) \models^4_{MKNF} \varphi$. We denote program of the form (3) corresponding to $\mathcal{P}$ by P. First we have a conclusion (†): $\mathcal{M}$ is a paraconsistent MKNF model of $\mathcal{K}$ iff I is the p-stable model of program P, where $I = \bigcap_{\mathcal{I}\in\mathcal{M}} \mathcal{M}$. And then $\pi(\mathcal{P}) \models^4_{MKNF} \varphi$ infers that for every paraconsistent MKNF model $\mathcal{M}$ of $\pi(\mathcal{P})$, $\mathcal{M} \models_4 \varphi$. Then we have $I \models_4 \varphi$, where $I = \bigcap_{\mathcal{I}\in\mathcal{M}} \mathcal{M}$. From (†), we can infer that the p-stable model I of program P is one-to-one corresponding to the paraconsistent MKNF model $\mathcal{M}$ of $\mathcal{P}$. Then we obtain that $P \models_4 \varphi$. The converse can be proved similarly.

Proof of (†): For every rule r in P^I, $\{L_{m+1}, \dots, L_n\} \cap I = \emptyset$. if $\{L_{l+1}, \dots, L_m\} \subseteq I$, then $\{L_{l+1}, \dots, L_m\} \subseteq \bigcap_{\mathcal{I}\in\mathcal{M}} \mathcal{M}$, and then $\mathcal{M} \models_4 \mathbf{K}L_i,\ l+1 \ \leq i \leq \ m$. Since $\mathcal{M}$ is the paraconsistent MKNF model of $\mathcal{P}$, we can infer that there exists a j, $1 \leq j \leq l, \mathcal{M} \models_4 \mathbf{K}L_j$, then we obtain that $L_j \in I$ and I satisfies every rule in P^I. Next we prove the minimality of I. Suppose there exists a interpretation J, such that $J \subset I$ and J satisfies P^I. Let $\mathcal{M}' = \{\mathcal{J} \mid \mathcal{J} \supseteq J\}$, then $\mathcal{M} \subset \mathcal{M}'$. For every MKNF rule r, if $\mathcal{M}' \models_4 \mathbf{not}L_j, m+1 \leq j \leq k$, and $\mathcal{M}' \models_4 \mathbf{K}L_i, l+1 \leq i \leq m$, which mean that $L_j \notin J, m+1 \leq j \leq k$, and $L_i \in J, l+1 \leq i \leq m$, then we imply there exists $i, 1 \leq i \leq l$, such that $L_i \in J$, since J satisfies P^I. Thus $\mathcal{M}' \models_4 \mathbf{K}L_i$. This contradicts with the fact that $\mathcal{M}$ is the paraconsistent MKNF model of $\mathcal{K}$. Then I is the p-stable model of P.

The sufficiency can be proved similarly. We omit the details here. □

For $\mathcal{K}$ = ($\mathcal{O}$,$\mathcal{P}$), Let $\mathcal{P}_G$ be the set of rules obtained from $\mathcal{P}$ by replacing in each rule all variables with all constants from $\mathcal{K}$ in all possible ways; the knowledge base $\mathcal{K}_G$ = ($\mathcal{O}$,$\mathcal{P}_G$) is called the *ground instantiation of* $\mathcal{K}$.

Lemma 1 *Let $\mathcal{K}_G$ be the ground instantiation of a hybrid MKNF knowledge base $\mathcal{K}$. Then the paraconsistent MKNF models of $\mathcal{K}_G$ and $\mathcal{K}$ coincide.*

Therefore, in the remainder of the paper we consider only grounding knowledge bases.

Example 1 *Let $\mathcal{K}_G^e = (\mathcal{O}^e, \mathcal{P}_G^e)$ be a hybrid MKNF knowledge base, where $\mathcal{O}^e = \{R \sqsubseteq \neg P, R(a)\}$ and $\mathcal{P}^e = \{\mathbf{K}P(a) \leftarrow \mathbf{not}P(a)\}$.*

Note that $\mathcal{O}^e$ and $\mathcal{P}^e$ are consistent knowledge bases, respectively. However, the combination causes inconsistency of $P(a)$. In this case, we will lose some useful information with the original reasoner of Hybrid MKNF knowledge base, for example, $R(a)$. Under the paraconsistent semantics, we can obtain a paraconsistent MKNF model $\mathcal{M}$ of $\mathcal{K}_G^e$; $\mathcal{M} = \{\mathcal{I} \mid \mathcal{I} \models_4 R(a) \wedge P(a) \wedge \neg P(a)$. Thus we can also infer $\mathcal{K}_G^e \models_{MKNF}^4 R(a)$.

4 Transformation from Paraconsistent Semantics to the Stable Model Semantics

In this section, we present a paraconsistent reasoning approach with hybrid knowledge bases. It is based on a transformation operator from the paraconsistent semantics to the stable model semantics.[1]

Given a hybrid MKNF knowledge base $\mathcal{K} = (\mathcal{O}, \mathcal{P})$, the transformation operator λ assigns to every MKNF formula φ some $\lambda(\varphi)$, where $\lambda(\varphi)$ is also an MKNF formula. The transformation is defined inductively as follows:

- If $\varphi = P(t_1, \ldots, t_l)$, then $\lambda(\varphi) = P_+(t_1, \ldots, t_l)$, where $P(t_1, \ldots, t_l)$ is a first-order atom occurring in $\mathcal{K}$ and $P_+(t_1, \ldots, t_l)$ is a new first-order atom;
- If $\varphi = \neg P(t_1, \ldots, t_l)$, then $\lambda(\varphi) = P_-(t_1, \ldots, t_l)$, where $P_-(t_1, \ldots, t_l)$ is a new first-order atom;
- If $\varphi = \varphi_1 \wedge \varphi_2$, then $\lambda(\varphi) = \lambda(\varphi_1) \wedge \lambda(\varphi_2)$, where φ_1 and φ_1 are two MKNF formulae;
- If $\varphi = \exists x : \psi$, then $\lambda(\varphi) = \exists x : \lambda(\psi)$;
- If $\varphi = \varphi_1 \supset \varphi_2$, then $\lambda(\varphi) = \lambda(\varphi_1) \supset \lambda(\varphi_2)$;
- If $\varphi = \varphi_1 \vee \varphi_2$, then $\lambda(\varphi) = \lambda(\varphi_1) \vee \lambda(\varphi_2)$;
- If $\varphi = \forall x : \psi$, then $\lambda(\varphi) = \forall x : \lambda(\psi)$;
- If $\varphi = \varphi_1 \equiv \varphi_2$, then $\lambda(\varphi) = \lambda(\varphi_1) \equiv \lambda(\varphi_2)$;
- If $\varphi = \mathbf{K}\psi$, then $\lambda(\varphi) = \mathbf{K}\lambda(\psi)$;
- If $\varphi = \mathbf{not}\psi$, then $\lambda(\varphi) = \mathbf{not}\lambda(\psi)$;
- If $\varphi = \neg(\varphi_1 \wedge \varphi_2)$, then $\lambda(\varphi) = \lambda(\neg\varphi_1) \vee \lambda(\neg\varphi_2)$;
- If $\varphi = \neg(\varphi_1 \vee \varphi_2)$, then $\lambda(\varphi) = \lambda(\neg\varphi_1) \wedge \lambda(\neg\varphi_2)$;
- If $\varphi = \neg(\exists x : \psi)$, then $\lambda(\varphi) = \forall x : \lambda(\neg\psi)$;
- If $\varphi = \neg(\forall x : \psi)$, then $\lambda(\varphi) = \exists x : \lambda(\neg\psi)$.

Then the hybrid MKNF knowledge base $\mathcal{K}$ is transformed inductively to a new hybrid MKNF KBs, denoted by $\overline{\mathcal{K}}$. Typically we assume the DL-part of $\mathcal{K}$ contains two types of axioms: $C \sqsubseteq D$ and $C(a)$. Then the transformed hybrid MKNF knowledge base $\overline{\mathcal{K}}$ consists of axioms and MKNF rules of the following three types: $\lambda(C) \sqsubseteq \lambda(D)$, $\lambda(C)(a)$ and $\lambda(\mathbf{K}H_1) \vee \ldots \vee \lambda(\mathbf{K}H_n) \leftarrow \lambda(\mathbf{K}A_{n+1}) \wedge \ldots \wedge \lambda(\mathbf{K}A_m), \lambda(\mathbf{not}B_{m+1}) \wedge \ldots \wedge \lambda(\mathbf{not}B_k)$. We say $\overline{\mathcal{K}}$ is classically

[1] The transformation operator has been introduced in [18] for OWL.

induced by a hybrid MKNF knowledge base $\mathcal{K}$, if all axioms and rules in $\overline{\mathcal{K}}$ are exactly the transformations of axioms and rules in $\mathcal{K}$.

The interpretation of $\overline{\mathcal{K}}$ can be induced by paraconsistent interpretation of $\mathcal{K}$. First of all, we define the interpretation structure.

Definition 11 (Classical Induced MKNF Structure) *Let $(\mathcal{I}, \mathcal{M}, \mathcal{N})$ be a paraconsistent MKNF structure of $\mathcal{K}$, and $\overline{\mathcal{K}}$ be the classical induced MKNF knowledge base of $\mathcal{K}$. The classical induced MKNF structure of $(\mathcal{I}, \mathcal{M}, \mathcal{M})$, written $(\overline{\mathcal{I}}, \overline{\mathcal{M}}, \overline{\mathcal{N}})$, is defined as follows:*

1. $\triangle = \overline{\triangle}$;
2. *for a first-order atom* $P(t_1, \ldots, t_l)$,

$$P_+{}^{\overline{\mathcal{I}}}(t_1, \ldots, t_l) = \begin{cases} \mathbf{t} & \textit{iff } P^{\mathcal{I}}(t_1, \ldots, t_l) \in \{\mathbf{t}, \top\} \\ \mathbf{f} & \textit{iff } P^{\mathcal{I}}(t_1, \ldots, t_l) \in \{\mathbf{f}, \bot\} \end{cases}$$

$$P_-{}^{\overline{\mathcal{I}}}(t_1, \ldots, t_l) = \begin{cases} \mathbf{t} & \textit{iff } \neg P^{\mathcal{I}}(t_1, \ldots, t_l) \in \{\mathbf{t}, \top\} \\ \mathbf{f} & \textit{iff } \neg P^{\mathcal{I}}(t_1, \ldots, t_l) \in \{\mathbf{f}, \bot\} \end{cases}$$

3. $\overline{\mathcal{M}}$ *and* $\overline{\mathcal{N}}$ *are nonempty sets of above defined* $\overline{\mathcal{I}}$.

Conversely, given an MKNF structure of a hybrid MKNF knowledge base $\mathcal{K}$, we can define the four-valued induced MKNF structure of $\mathcal{K}$ easily. Particularly, when $\mathcal{K}$ is consistent, the four-valued induced MKNF structure coincides with the original MKNF structure.

Definition 12 (Four-valued Induced MKNF Structure) *Let $(\overline{\mathcal{I}}, \overline{\mathcal{M}}, \overline{\mathcal{N}})$ be a MKNF structure of a hybrid MKNF knowledge base $\mathcal{K}$. The four-valued induced MKNF structure of it, written $(\mathcal{I}, \mathcal{M}, \mathcal{N})$, is defined as follows:*

1. $\triangle = \overline{\triangle}$;
2. *for a first-order atom* $P(t_1, \ldots, t_l)$,

$$P^{\mathcal{I}}(t_1, \ldots, t_l) = \begin{cases} \mathbf{t} & \textit{iff } P_+(t_1, \ldots, t_l) \in \overline{\mathcal{I}} \textit{ and } P_-(t_1, \ldots, t_l) \notin \overline{\mathcal{I}} \\ \mathbf{f} & \textit{iff } P_+(t_1, \ldots, t_l) \notin \overline{\mathcal{I}} \textit{ and } P_-(t_1, \ldots, t_l) \in \overline{\mathcal{I}} \\ \top & \textit{iff } P_+(t_1, \ldots, t_l) \in \overline{\mathcal{I}} \textit{ and } P_-(t_1, \ldots, t_l) \in \overline{\mathcal{I}} \\ \bot & \textit{iff } P_+(t_1, \ldots, t_l) \notin \overline{\mathcal{I}} \textit{ and } P_-(t_1, \ldots, t_l) \notin \overline{\mathcal{I}} \end{cases}$$

3. $\mathcal{M}$ *and* $\mathcal{N}$ *are nonempty sets of above defined* $\mathcal{I}$.

Lemma 2 *For a paraconsistent MKNF structure $(\mathcal{I}, \mathcal{M}, \mathcal{N})$ of a hybrid MKNF knowledge base $\mathcal{K}$ and any MKNF formulae φ, we have*

$$(\overline{\mathcal{I}}, \overline{\mathcal{M}}, \overline{\mathcal{N}})\overline{\varphi} = \begin{cases} \mathbf{t} & \textit{iff } (\mathcal{I}, \mathcal{M}, \mathcal{N})\varphi \in \{\mathbf{t}, \top\} \\ \mathbf{f} & \textit{iff } (\mathcal{I}, \mathcal{M}, \mathcal{N})\varphi \in \{\mathbf{f}, \bot\} \end{cases} \tag{4}$$

$$(\overline{\mathcal{I}}, \overline{\mathcal{M}}, \overline{\mathcal{N}})\overline{\neg\varphi} = \begin{cases} \mathbf{t} & \textit{iff } (\mathcal{I}, \mathcal{M}, \mathcal{N})\neg\varphi \in \{\mathbf{t}, \top\} \\ \mathbf{f} & \textit{iff } (\mathcal{I}, \mathcal{M}, \mathcal{N})\neg\varphi \in \{\mathbf{f}, \bot\} \end{cases} \tag{5}$$

Proof. We prove the two equations (4) and (5) by induction. For simplicity, we give the proof details of some forms of formulae.
Case 1: $\psi = \varphi_1 \wedge \varphi_2$.

From the induction assumption, φ_1 and φ_2 satisfy equations (4) and (5). That is to say,

$$(\overline{\mathcal{I}}, \overline{\mathcal{M}}, \overline{\mathcal{N}})\overline{\varphi_i} = \begin{cases} \mathbf{t} & \text{iff } (\mathcal{I}, \mathcal{M}, \mathcal{N})\varphi_i \in \{\mathbf{t}, \top\} \\ \mathbf{f} & \text{iff } (\mathcal{I}, \mathcal{M}, \mathcal{N})\varphi_i \in \{\mathbf{f}, \bot\} \end{cases}$$

where $i = 1, 2$. Clearly we have

$$(\overline{\mathcal{I}}, \overline{\mathcal{M}}, \overline{\mathcal{N}})\overline{(\varphi_1 \ \wedge \ \varphi_2)} = \begin{cases} \mathbf{t} & \text{iff } (\mathcal{I}, \mathcal{M}, \mathcal{N})(\varphi_1 \wedge \varphi_2) \in \{\mathbf{t}, \top\} \\ \mathbf{f} & \text{iff } (\mathcal{I}, \mathcal{M}, \mathcal{N})(\varphi_1 \wedge \varphi_2) \in \{\mathbf{f}, \bot\} \end{cases}$$

Therefore when $\psi = \varphi_1 \wedge \varphi_2$, the equations hold.
Case 2: $\psi = \mathbf{K}\varphi$.

From the induction assumption, φ satisfies equations (4) and (5). $(\overline{\mathcal{I}}, \overline{\mathcal{M}}, \overline{\mathcal{N}})$ $\overline{\mathbf{K}\varphi} = \mathbf{t}$ iff $(\overline{\mathcal{J}}, \overline{\mathcal{M}}, \overline{\mathcal{N}})\overline{\varphi} = \mathbf{t}$ for all $\overline{\mathcal{J}} \in \overline{\mathcal{M}}$. From assumption, $(\overline{\mathcal{J}}, \overline{\mathcal{M}}, \overline{\mathcal{N}})\overline{\varphi} = \mathbf{t}$ for all $\overline{\mathcal{J}} \in \overline{\mathcal{M}}$ iff $(\mathcal{J}, \mathcal{M}, \mathcal{N})\varphi \in \{\mathbf{t}, \top\}$ for all $\mathcal{J} \in \mathcal{M}$, which means that $(\mathcal{I}, \mathcal{M}, \mathcal{N})\mathbf{K}\varphi \in \{\mathbf{t}, \top\}$. Then we obtain $(\overline{\mathcal{I}}, \overline{\mathcal{M}}, \overline{\mathcal{N}})\overline{\mathbf{K}\varphi} = \mathbf{t}$ iff $(\mathcal{I}, \mathcal{M}, \mathcal{N})\mathbf{K}\varphi \in \{\mathbf{t}, \top\}$. Similarly, we can obtain equation (4) easily. Here we omit the details.
Case 3: $\psi = \mathbf{not}\varphi$.

From the induction assumption, φ satisfies equations (4) and (5). $(\overline{\mathcal{I}}, \overline{\mathcal{M}}, \overline{\mathcal{N}})$ $\overline{\mathbf{not}\varphi} = \mathbf{t}$ iff $(\overline{\mathcal{J}}, \overline{\mathcal{M}}, \overline{\mathcal{N}})\overline{\varphi} = \mathbf{f}$ for some $\overline{\mathcal{J}} \in \overline{\mathcal{M}}$. From assumption, $(\mathcal{J}, \mathcal{M}, \mathcal{N})\varphi \in \{\mathbf{f}, \bot\}$ for some $\mathcal{J} \in \mathcal{M}$, which means that $(\mathcal{I}, \mathcal{M}, \mathcal{N})\mathbf{not}\varphi \in \{\mathbf{t}, \top\}$ from definition 7. Therefore we obtain that $(\overline{\mathcal{I}}, \overline{\mathcal{M}}, \overline{\mathcal{N}})\overline{\mathbf{not}\varphi} = \mathbf{t}$ iff $(\mathcal{I}, \mathcal{M}, \mathcal{N})\mathbf{not}\varphi \in \{\mathbf{t}, \top\}$. Similarly, equation (5) can be obtained.

The case for MKNF formulae of other forms in $\mathcal{K}$ can be proved in the same way. $\square$

From Lemma 2, we can get an important conclusion as follow:

Theorem 1 *For a hybrid MKNF knowledge base* $\mathcal{K}$ *and an MKNF formula* φ*, we have* $\mathcal{K} \models^4_{MKNF} \varphi$ *iff* $\overline{\mathcal{K}} \models_{MKNF} \overline{\varphi}$*.*

Proof. (Necessity) Suppose $\mathcal{M}$ is a paraconsistent MKNF model of $\mathcal{K}$, we only need to prove the classical induction interpretation $\overline{\mathcal{M}}$ of $\mathcal{M}$ is the MKNF model of $\overline{\mathcal{K}}$. From $\mathcal{M} \models^4_{MKNF} \pi(\mathbf{K})$, we obtain that $\mathcal{M} \models^4_{MKNF} \mathbf{K}\pi(\mathcal{O})$ and $\mathcal{M} \models^4_{MKNF} \pi(\mathcal{P})$. Typically, let $\mathcal{O}$ consists of DL axioms of forms C(a) and $C \sqsubseteq D$, and MKNF rules are of form (2).

Case 1: For axioms of form C(a), it is the atom MKNF formula and easy to infer that $(\overline{\mathcal{I}}, \overline{\mathcal{M}}, \overline{\mathcal{M}})\overline{\mathbf{K}C(a)} = \mathbf{t}$ iff $(\mathcal{I}, \mathcal{M}, \mathcal{M})\mathbf{K}C(a) \in \{\mathbf{t}, \top\}$ from Lemma 2. Then we obtain $(\overline{\mathcal{I}}, \overline{\mathcal{M}}, \overline{\mathcal{M}}) \models \overline{C(a)}$. If there exists a MKNF interpretation $\overline{\mathcal{M}'} \supset \overline{\mathcal{M}}$ such that $(\overline{\mathcal{I}'}, \overline{\mathcal{M}'}, \overline{\mathcal{M}})\overline{\mathbf{K}C(a)} = \mathbf{t}$, then from Lemma 2, the four-valued induction structure $(\mathcal{I}', \mathcal{M}', \mathcal{M})\mathbf{K}C(a) = \mathbf{t}$ *or* $\top$, contradiction.

Case 2: For axioms of form $C \sqsubseteq D$, from [20], $\pi(C \sqsubseteq D) = \forall x : (C(x) \rightarrow D(x))$. From Lemma 2, $(\overline{\mathcal{I}}, \overline{\mathcal{M}}, \overline{\mathcal{M}})\overline{\mathbf{K}(\forall x : (C(x) \rightarrow D(x)))} = \mathbf{t}$ iff $(\mathcal{I}, \mathcal{M}, \mathcal{M})\mathbf{K}(\forall x :$

$(C(x) \rightarrow D(x))) \in \{\mathbf{t}, \top\}$, in $\overline{\text{which the later holds from}}$ assumption. Then the former holds also. Therefore, $\overline{M} \models_{MKNF} \overline{\mathbf{K}(\pi(C \sqsubseteq D))}$.

Case 3: For MKNF rules of form (1), a paraconsistent MKNF structure ($\mathcal{I}$, $\mathcal{M}$, $\mathcal{M}$) paraconsistently satisfies a MKNF rule r iff it paraconsistently satisfies an element $\mathbf{K}H_i$ in the rule head, or does't paraconsistently satisfies some element in the rule body that may be the form of $\mathbf{K}A_j$ or $\mathbf{not}B_t$. In this case the classical induced structure $(\overline{\mathcal{I}}, \overline{\mathcal{M}}, \overline{\mathcal{M}})$ satisfies $\overline{\mathbf{K}H_i}$ or do not satisfies $\overline{\mathbf{K}A_j}$ or $\overline{\mathbf{not}B_t}$, which means $(\overline{\mathcal{I}}, \overline{\mathcal{M}}, \overline{\mathcal{M}})$ satisfies the induced MKNF rule $\overline{r}$.

If there exists a MKNF interpretation $\overline{\mathcal{M}'} \supset \overline{\mathcal{M}}$ such that $(\overline{\mathcal{I}'}, \overline{\mathcal{M}'}, \overline{\mathcal{M}})\overline{\pi(\mathcal{K})} = \mathbf{t}$, then this MKNF interpretation structure must satisfies all the induced axioms and rules, then from Lemma 2 and the maximality of $\mathcal{M}$, we can obtain contradiction in the way present in case 1.
(Sufficiency) Similarly we can prove this using Lemma 2 via the same method as above. $\square$

Note that the transformation operator is linear. Thus from Theorem 1, we can conclude that the data complexity of our paradigm is not higher than that of classical reasoning.

5 Characterization of Paraconsistent MKNF Models

In this section we present a fixpoint characterization of paraconsistent MKNF models of a hybrid MKNF knowledge base $\mathcal{K} = (\mathcal{O},\mathcal{P})$. According to the paraconsistent semantics, $\mathcal{K}$ has exactly the same paraconsistent MKNF model as $\mathcal{K}_G$. Therefore in the rest of the paper, we only consider grounded knowledge bases $\mathcal{K}_G$.

5.1 Positive Rules

A positive MKNF rule has the form

$$\mathbf{K}H_1 \lor \ldots \lor \mathbf{K}H_n \leftarrow \mathbf{K}A_1 \land \ldots \land \mathbf{K}A_m$$

where H_i and A_i are literals occurring in $\mathcal{K}_G$.

Definition 13 *Let $\mathcal{K}_G$ be a ground hybrid MKNF knowledge base. The set of $\mathbf{K}$-atoms of $\mathcal{K}_G$, written $\mathsf{KA}(\mathcal{K}_G)$, is the smallest set that contains (1) all ground $\mathbf{K}$-atoms occurring in $\mathcal{P}_G$, and (2) a modal atom $\mathbf{K}\xi$ for each ground modal atom $\mathbf{not}\xi$ occurring in $\mathcal{P}_G$. Furthermore, $\mathsf{HA}(\mathcal{K}_G)$ is the subset of $\mathsf{KA}(\mathcal{K}_G)$ that contains all $\mathbf{K}$-atoms occurring in the head of some rule in $\mathcal{P}_G$.*

As argued in [19], MKNF models of $\mathcal{K}_G$ are decided by subsets of $\mathsf{HA}(\mathcal{K}_G)$. The same holds for paraconsistent MKNF models.

Definition 14 *Let $\mathcal{K}_G$ be a ground hybrid MKNF knowledge base, and P_h a subset of $\mathsf{HA}(\mathcal{K}_G)$. The objective knowledge of P_h w.r.t. $\mathcal{K}_G$ is the first-order theory $\mathsf{OB}_{\mathcal{O},P_h}$ defined by*

$$\mathsf{OB}_{\mathcal{O},P_h} = \{\pi(\mathcal{O})\} \cup \{\ \xi\ \mid \mathbf{K}\ \xi \in P_h\}.$$

Definition 15 *For a paraconsistent MKNF interpretation $\mathcal{M}$ and a set of ground* $\mathbf{K}$*-atoms S, the subset of S paraconsistently induced by $\mathcal{M}$ is the set* $\{\mathbf{K}\ \xi \in S \mid \mathcal{M} \models_4 \xi\}$.

Lemma 3 *Let $\mathcal{K}_G$ = ($\mathcal{O}$,$\mathcal{P}_G$) be a ground hybrid MKNF knowledge base, $\mathcal{M}$ a paraconsistent MKNF model of $\mathcal{K}_G$, and P_h the subset of $\mathcal{P}_G$ paraconsistently induced by $\mathcal{M}$. Then $\mathcal{M}$ coincides with the set of paraconsistent MKNF interpretation* $\mathcal{M}' = \{\mathcal{I} \mid \mathcal{I} \models_4 \mathit{OB}_{\mathcal{O},P_h}\}$.

The proof is similar to Lemma 4.4 in [19].

Next all we have to do is search for appropriate P_h. In the positive case, we define a fixpoint operator to evaluate P_h.

Definition 16 *Let $\mathcal{K}_G$ = ($\mathcal{O}$,$\mathcal{P}$) be a ground hybrid MKNF knowledge base and* $\mathbb{S} \in 2^{2^{\mathsf{HA}(\mathcal{K}_G)}}$. *A mapping* $\mathfrak{T}_{\mathcal{K}_G} : 2^{2^{\mathsf{HA}(\mathcal{K}_G)}} \to 2^{2^{\mathsf{HA}(\mathcal{K}_G)}}$ *is defined as*

$$\mathfrak{T}_{\mathcal{K}_G}(\mathbb{S}) = \bigcup_{\mathtt{S} \in \mathbb{S}} T_{\mathcal{K}_G}(\mathtt{S}),$$

where the mapping $T_{\mathcal{K}_G} : 2^{\mathsf{HA}(\mathcal{K}_G)} \to 2^{2^{\mathsf{HA}(\mathcal{K}_G)}}$ *is defined as follows.*

- ◇ *If* $\{\mathbf{K}A_1, \ldots, \mathbf{K}A_m\} \subseteq \mathtt{S}$ *for some ground integrity constraint* $\leftarrow \mathbf{K}A_1 \wedge \ldots \wedge \mathbf{K}A_m$ *in $\mathcal{P}$, then* $T_{\mathcal{K}_G}(\mathtt{S}) = \emptyset$;
- ◇ *Otherwise,* $T_{\mathcal{K}_G}(\mathtt{S}) = \{\mathtt{Q}_i \mid$ *for each ground MKNF rule* C_j : $\mathbf{K}H_1 \vee \ldots \vee \mathbf{K}H_n \leftarrow \mathbf{K}A_1 \wedge \ldots \wedge \mathbf{K}A_m$ *such that* $\{\mathbf{K}A_1, \ldots , \mathbf{K}A_m\} \subseteq \mathtt{S}, \mathtt{Q}_i = \mathtt{S} \cup \bigcup_j \mathbf{K}H_i, 1 \leq i \leq n\}$.

Lemma 4 $\mathfrak{T}_{\mathcal{K}_G}$ *is a monotonic operator on* $2^{2^{\mathsf{HA}(\mathcal{K}_G)}}$.

Proof. Suppose $\mathbb{S}_1 \subseteq \mathbb{S}_2 \subseteq 2^{\mathsf{HA}(\mathcal{K}_G)}$, and $\mathbb{S}_0 = \mathbb{S}_2 / \mathbb{S}_1$. $\mathfrak{T}_{\mathcal{K}_G}(\mathbb{S}_2) = \bigcup_{\mathtt{S} \in \mathbb{S}_2} T_{\mathcal{K}_G}(\mathtt{S}) = \mathfrak{T}_{\mathcal{K}_G}(\mathbb{S}_1) \cup \bigcup_{\mathtt{S} \in \mathbb{S}_0} T_{\mathcal{K}_G}(\mathtt{S})$. Clearly, $\mathfrak{T}_{\mathcal{K}_G}(\mathbb{S}_1) \subseteq \mathfrak{T}_{\mathcal{K}_G}(\mathbb{S}_2)$. Thus, $\mathfrak{T}_{\mathcal{K}_G}$ is monotonic. □

From Lemma 4, we can get a least fixpoint of $\mathfrak{T}_{\mathcal{K}_G}$ by the following procedure:

$$\mathfrak{T}_{\mathcal{K}_G} \uparrow 0 = \emptyset$$

$$\mathfrak{T}_{\mathcal{K}_G} \uparrow n+1 = \mathfrak{T}_{\mathcal{K}_G} \mathfrak{T}_{\mathcal{K}_G} \uparrow n$$

$$\mathfrak{T}_{\mathcal{K}_G} \uparrow \omega = \bigcup_{\alpha < \omega} \bigcap_{\alpha < n < \omega} \mathfrak{T}_{\mathcal{K}_G} \uparrow n$$

For a finite ground program $\mathcal{P}_G$, the fixpoint of the operator $\mathfrak{T}_{\mathcal{K}_G}$ is $\mathfrak{T}_{\mathcal{K}_G} \uparrow n$. If $\mathcal{P}_G$ is infinite, the fixpoint is $\mathfrak{T}_{\mathcal{K}_G} \uparrow \omega$.

Lemma 5 *Let $\mathcal{K}_G$ = ($\mathcal{O}$,$\mathcal{P}_G$) be a hybrid MKNF knowledge base, and P_h the set paraconsistently induced by a paraconsistent MKNF interpretation $\mathcal{M}$ of $\mathcal{K}_G$. Then $\mathcal{M} \models_4 \mathcal{K}_G$ if and only if* $P_h \in \mathfrak{T}_{\mathcal{K}_G}(\{P_h\})$.

Proof. (Sufficiency) If $P_h \in \mathfrak{T}_{\mathcal{K}_G}(\{P_h\})$, then $\{\mathbf{K}A_1, \ldots, \mathbf{K}A_m\} \subseteq P_h$ implies that there exists a rule head $\mathbf{K}H_i$ such that in $\mathbf{K}H_i \in P_h$. Since P_h is paraconsistently induced by $\mathcal{M}$, we infer if $\mathcal{M} \models_4 \mathbf{K}A_j, 1 \leq j \leq m$. Then $\mathcal{M} \models_4 \mathbf{K}H_i$. Therefore, $\mathcal{M} \models_4 \mathcal{K}_G$.
(Necessity) If $\mathcal{M} \models_4 \mathcal{K}_G$, then $\mathcal{M}$ paraconsistently satisfies all the MKNF rules in program $\mathcal{P}$. If $\mathcal{M} \models_4 \mathbf{K}A_j, 1 \leq j \leq m$, which means $\{\mathbf{K}A_1, \ldots, \mathbf{K}A_m\} \subseteq P_h$, then there exist $i, 1 \leq i \leq n$, $\mathcal{M} \models_4 \mathbf{K}H_i$, which means that $\mathbf{K}H_i \in P_h$. Therefore $P_h \in \mathfrak{T}_{\mathcal{K}_G}(\{P_h\})$. $\square$

$P_h \in \mathfrak{T}_{\mathcal{K}_G}(\{P_h\})$ indicates that implication "$\leftarrow$" is closed w.r.t. P_h. That is to say, if $\{\mathbf{K}A_1, \ldots \mathbf{K}A_m\} \subseteq P_h$, then there must exists a rule head $\mathbf{K}H_i$ in P_h, which is equivalent to $\mathcal{M} \models_4 \mathcal{P}_G$. With Lemma 5, we obtain the P_h corresponding to the paraconsistent MKNF model of $\mathcal{K}_G$.

Let $\gamma(\mathfrak{T}_{\mathcal{K}_G} \uparrow \omega) = \{\mathtt{S} \mid \mathtt{S} \in \mathfrak{T}_{\mathcal{K}_G} \uparrow \omega, \textit{and}\ \mathtt{S} \in \mathfrak{T}_{\mathcal{K}_G}(\{\mathtt{S}\})\}$, and $min(\mathbb{S}) = \{\mathtt{S} \mid \textit{ there exists no } \mathtt{Q} \in \mathbb{S} \textit{ such that } \mathtt{Q} \subset \mathtt{S}\}$.

Theorem 2 *Let $\mathcal{K}_G = (\mathcal{O}, \mathcal{P}_G)$ be a ground hybrid MKNF knowledge base and $\mathcal{M}$ a paraconsistent MKNF model of $\mathcal{K}_G$. Then $\mathcal{M}$ coincides with $\mathcal{M}' = \{\mathcal{I} \mid \mathcal{I} \models_4 OB_{\mathcal{O},P_h}\}$, where P_h is an element of the set $\mathbb{Q} = min(\gamma(\mathfrak{T}_{\mathcal{K}_G} \uparrow \omega))$.*

Proof. From Lemma 5, $\mathcal{M} \models_4 \mathcal{K}_G$. Next thing is to prove the maximality of $\mathcal{M}$. Suppose there exists a $\mathcal{M}' \supsetneq \mathcal{M}$ such that $(\mathcal{I}', \mathcal{M}', \mathcal{M}) \models_4 \mathcal{K}_G$ for each $\mathcal{I}' \in \mathcal{M}'$. Let P'_h be the set induced by $\mathcal{M}'$, and $\mathcal{M}'' = \{\mathcal{I} \mid \mathcal{I} \models_4 \mathsf{OB}_{\mathcal{O},P'_h}\}$. Then $(\mathcal{I}'', \mathcal{M}'', \mathcal{M}) \models_4 \mathcal{K}_G$ for each $\mathcal{I}'' \in \mathcal{M}''$. We denote this property by $(\natural)$, From Lemma 5, $(\mathcal{I}'', \mathcal{M}'', \mathcal{M}) \models_4 \mathcal{K}_G$ implies $P'_h \in \mathfrak{T}_{\mathcal{K}_G}(\{P'_h\})$. Then $P'_h \in min(\gamma(\mathfrak{T}_{\mathcal{K}_G} \uparrow \omega))$, which contradict with the minimality of with P_h. Therefore the theory hold. $\square$

Proposition 3 *Let $\mathcal{K}_G = (\mathcal{O}, \mathcal{P}_G)$ be a ground hybrid MKNF knowledge base. $\mathcal{P}_G$ is a positive nondisjunctive program. If $\mathcal{K}_G$ has a paraconsistent MKNF model, then $\mathcal{M}' = \{\mathcal{I} \mid \mathcal{I} \models_4 OB_{\mathcal{O},P_h}\}$, where $P_h \in \mathbb{Q} = min(\gamma(\mathfrak{T}_{\mathcal{K}_G} \uparrow \omega))$ and P_h is unique. That is to say, the set $\mathbb{Q}$ has only one element.*

5.2 General Rules

In this section, we characterize the paraconsistent MKNF model of $\mathcal{K}_G$ using the fixpoint operator presented in the previous section. We first introduce a program transformation which translate the general program to a positive program.[2]

Definition 17 *Let $\mathcal{K}_G = (\mathcal{O}, \mathcal{P}_G)$ be a ground hybrid MKNF knowledge base. Then its transformation is defined as $\mathcal{K}_G^*$ obtained by replacing each MKNF rule of Definition 5 in $\mathcal{P}_G$ with the following positive MKNF rules.*

$$\mathbf{K}\mu_1 \vee \ldots \vee \mathbf{K}\mu_n \vee \mathbf{K}B'_{m+1} \vee \ldots \vee \mathbf{K}B'_k \leftarrow \mathbf{K}A_1 \wedge \ldots \wedge \mathbf{K}A_m, \tag{6}$$

$$\mathbf{K}H_i \leftarrow \mathbf{K}\mu_i \quad for\ 1 \leq i \leq n, \tag{7}$$

[2] This method is inspired by [23].

$$\leftarrow \mathbf{K}\mu_i \wedge \mathbf{K}B_j \quad \textit{for } 1 \leq i \leq n, m+1 \leq j \leq k, \tag{8}$$

$$\mathbf{K}\mu_i \leftarrow \mathbf{K}H_i \quad \textit{for } 1 \leq i \leq n. \tag{9}$$

Let $\mathtt{S}^*$ be the subset of $\mathsf{HA}(\mathcal{K}_G^*)$, called *canonical*, if $\mathbf{K}B_i' \in \mathtt{S}^*$ implies $\mathbf{K}B_i \in \mathtt{S}^*$, and vice versa. Given a set $\mathbb{S}^*$ that is a subset of $2^{\mathsf{HA}(\mathcal{K}_G^*)}$, $\Phi(\mathbb{S}^*) = \{\mathtt{S}^* \cap \mathsf{HA}(\mathcal{K}_G) \mid \mathtt{S}^* \in \mathbb{S}^* \textit{ and } \mathtt{S}^* \textit{ is canonical}\}$.

Theorem 3 *Let $\mathcal{K}_G = (\mathcal{O},\mathcal{P}_G)$ be a ground hybrid MKNF knowledge base and $\mathcal{M}$ a paraconsistent MKNF model of $\mathcal{K}_G$. Then $\mathcal{M}$ coincides with $\mathcal{M}' = \{\mathcal{I} \mid \mathcal{I} \models_4 \mathsf{OB}_{\mathcal{O},P_h}\}$, where P_h is an element of the set $\mathbb{Q} = \Phi(min(\gamma(\mathfrak{T}_{\mathcal{K}_G^*} \uparrow \omega)))$.*

Proof. Suppose that P_h is an element of the set $\mathbb{Q} = \Phi(min(\gamma(\mathfrak{T}_{\mathcal{K}_G^*} \uparrow \omega)))$, we need to prove that $\mathcal{M}' = \{\mathcal{I} \mid \mathcal{I} \models_4 \mathsf{OB}_{\mathcal{O},P_h}\}$ is the paraconsistent MKNF model of $\mathcal{K}_G$. Let $\mathtt{S}^*$ be the canonical subset of $\mathsf{HA}(\mathcal{K}_G^*)$ such that $\mathtt{S}^* \cap \mathsf{HA}(\mathcal{K}_G) = P_h$ and $\mathtt{S}^* \in min(\gamma(\mathfrak{T}_{\mathcal{K}_G^*} \uparrow \omega))$. For a MKNF rule r of form (2), if $\mathcal{M}' \models_4 \mathbf{K}A_i$, *where* $n+1 \leq i \leq m$, then $\mathbf{K}A_i \in P_h$, *where* $n+1 \leq i \leq m$. Since $P_h \subseteq \mathtt{S}^*$, we infer that $\mathbf{K}A_i \in \mathtt{S}^*$, *where* $n+1 \leq i \leq m$. This implies either (i). $\exists \mathbf{K}\mu_i \in \mathtt{S}^*$, $\mathbf{K}H_i \in P_h$, which infers $\exists \mathbf{K}H_i$ such that $\mathcal{M}' \models_4 \mathbf{K}H_i$ and $\{\mathbf{K}B_{m+1}, \ldots, \mathbf{K}B_k\} \cap \mathtt{S}^* = \emptyset$ that means $\{\mathbf{K}B_{m+1}, \ldots, \mathbf{K}B_k\} \cap P_h = \emptyset$ and for all $m+1 \leq j \leq k$, $\mathcal{M}' \models_4 \mathbf{not}B_j$. Therefore, $\mathcal{M}'$ paraconsistently satisfies the MKNF rule. Or (ii). $\exists \mathbf{K}B_i' \in \mathtt{S}^*$, and $\{\mathbf{K}H_1, \ldots, \mathbf{K}H_n\} \cap \mathtt{S}^* = \emptyset$. Since $\mathtt{S}^*$ is canonical, $\mathbf{K}B_i \in \mathtt{S}^*$, and then $\mathbf{K}B_i \in P_h$ and $\{\mathbf{K}H_1, \ldots, \mathbf{K}H_n\} \cap P_h = \emptyset$. Therefore, $\mathcal{M}'$ paraconsistently satisfies the MKNF rule also. Next we prove the maximality of $\mathcal{M}'$.

Suppose there exists a paraconsistent MKNF interpretation $\mathcal{M}'_1 \supset \mathcal{M}'$ such that $(\mathcal{I}', \mathcal{M}'_1, \mathcal{M}') \models_4 \mathcal{K}_G$. Let P_h' be the set paraconsistently induced by $\mathcal{M}'_1$, and $\mathcal{M}'_2 = \{\mathcal{I} \mid \mathcal{I} \models_4 \mathsf{OB}_{\mathcal{O},P_h'}\}$. Then we can infer that $(\mathcal{I}', \mathcal{M}'_2, \mathcal{M}') \models_4 \mathcal{K}_G$. Clearly, $P_h' \subset P_h$. Let $P_h \backslash P_h' = \{\mathbf{K}L\}$, and $\mathtt{S}_1^* = \mathtt{S}^* \setminus \{\mathbf{K}L\}$. If $\mathbf{K}A_i \in \mathtt{S}_1^*$, *where* $n+1 \leq i \leq m$, then $\mathbf{K}A_i \in P_h'$, *where* $n+1 \leq i \leq m$, since $\mathtt{S}_1^*$ is a subset of $\mathsf{HA}(\mathcal{K}_G^*)$. And then we can infer two conditions. When $\{\mathbf{K}B_{m+1}, \ldots, \mathbf{K}B_k\} \cap P_h' = \emptyset$, $\exists \mathbf{K}H_i \in P_h'$. Since $P_h' \subseteq \mathtt{S}_1^*$, therefore if $\{\mathbf{K}B_{m+1}, \ldots, \mathbf{K}B_k\} \cap \mathtt{S}_1^* = \emptyset$, then $\exists \mathbf{K}H_i \in \mathtt{S}_1^*$. Thus $\mathtt{S}_1^*$ paraconsistently satisfies $\mathcal{K}_G^*$. When $\exists \mathbf{K}B_i \in P_h'$. Since $(\mathcal{I}', \mathcal{M}'_2, \mathcal{M}') \models_4 \mathcal{K}_G$, we imply $\{\mathbf{K}H_1, \ldots, \mathbf{K}H_n\} \cap P_h' = \emptyset$, and $\{\mathbf{K}H_1, \ldots, \mathbf{K}H_n\} \cap \mathtt{S}_1^* = \emptyset$. Clearly $\mathtt{S}_1^*$ satisfies (6), (7), (8) and (9). This contradicts the fact that $\mathtt{S}^*$ is the minimal set in $\gamma(\mathfrak{T}_{\mathcal{K}_G^*} \uparrow \omega)$. Then $\mathcal{M}'$ is the paraconsistent MKNF model of $\mathcal{K}_G$.

Suppose $\mathcal{M}$ is the paraconsistent MKNF model of $\mathcal{K}_G$. From Lemma 3 we imply that $\mathcal{M} = \{\mathcal{I} \mid \mathcal{I} \models_4 \mathsf{OB}_{\mathcal{O},P_h}\}$, where P_h is the set paraconsistently induced by $\mathcal{M}$. For each MKNF rule, let $\mathtt{S}_\mu = \bigcup_r \{\mathbf{K}\mu_i \mid \mathbf{K}A_i \in P_h',\ \textit{where } n+1 \leq i \leq m, \{\mathbf{K}B_{m+1}, \ldots, \mathbf{K}B_k\} \cap P_h = \emptyset,\ \textit{and } \mathbf{K}H_i \in P_h\}$, and $\mathtt{S}_\vartheta = \bigcup_r \{\mathbf{K}B_i' \mid \mathbf{K}A_i \in P_h',\ \textit{where } n+1 \leq i \leq m,\ \textit{and } \mathbf{K}B_i \in P_h\}$. Let $\mathtt{S} = P_h \cup \mathtt{S}_\mu \cup \mathtt{S}_\vartheta$. Then, $\mathtt{S}$ satisfies all the MKNF rules in $\mathcal{K}_G^*$ and $\mathtt{S} \in \gamma(\mathfrak{T}_{\mathcal{K}_G^*} \uparrow \omega)$ by the construction. Now we define $\mathtt{S}^* = P_h \cup \mathtt{S}'$, in which $\mathtt{S}'$ is the minimal subset of $\mathtt{S}_\mu \cup \mathtt{S}_\vartheta$ such that each $\mathbf{K}B_i'$ and $\mathbf{K}\mu_i$ are chosen in a way that satisfies the MKNF rules in $\mathcal{K}_G^*$. Now we need to prove $\mathtt{S}^*$ to be canonical and minimal in $\gamma(\mathfrak{T}_{\mathcal{K}_G^*} \uparrow \omega)$. Canonical is obvious from the definition of $\mathtt{S}^*$. As to minimality, assume that

$\mathtt{S}^{*\prime} \in \gamma(\mathfrak{T}_{\mathcal{K}_G^*} \uparrow \omega)$ such that $\mathtt{S}^{*\prime} \subset \mathtt{S}^*$. Since $\mathtt{S}^*$ is defined to be the minimal set with respect to elements in $\mathtt{S}_\mu \cup \mathtt{S}_\vartheta$, then $\mathtt{S}^{*\prime} \cap \mathsf{HA}(\mathcal{K}_G) \subset \mathtt{S}^* \cap \mathsf{HA}(\mathcal{K}_G)$, and thus $\exists \mathbf{K}L \in \mathtt{S}^* \setminus \mathtt{S}^{*\prime}$. In this case, there exists a MKNF rule of form (6) in $\mathcal{K}_G^*$ such that $\{\mathbf{K}A_{n+1}, \ldots, \mathbf{K}A_m\} \subseteq \mathtt{S}^*$, $\mathbf{K}\mu_i \in \mathtt{S}^* \setminus \mathtt{S}^{*\prime}$, $\mathbf{K}B'_j \in \mathtt{S}^{*\prime}$ for some $1 \leq i \leq n$ and $m+1 \leq j \leq k$. Since $\mathtt{S}^{*\prime} \subset \mathtt{S}^*$, $\mathbf{K}B'_j \in \mathtt{S}^*$. $\mathtt{S}^*$ is canonical, which implies $\mathbf{K}B_j \in \mathtt{S}^*$. This is impossible from condition (6). Therefore, $\mathtt{S}^*$ is minimal in $\gamma(\mathfrak{T}_{\mathcal{K}_G^*} \uparrow \omega)$. The conclusion hold. □

6 Conclusion

In this paper we have presented a paraconsistent semantics of hybrid MKNF knowledge bases that is sound w.r.t. the classical two-valued semantics defined in [19], which restricts to the paraconsistent semantics of extended disjunctive program [23] and to the paraconsistent semantics of OWL [18], when the DL-part and LP-part is empty, respectively. Furthermore, we characterized paraconsistent MKNF models via a fixpoint operator, and showed that the complexity of our paradigm is not higher than that in [19].

There are a number of paths to further develop this work. In [13, 12], a well-founded semantics was introduced for hybrid MKNF knowledge bases which has better complexity properties, and our paraconsistent approach could be carried over to this paradigm. An even tighter paraconsistent and non-monotonic integration of OWL and rules could furthermore be investigated following the ideas from [14, 15].

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