Exercise Sheet 10
CS 2210 Logic for Computer Scientists (Hitzler)
Solutions due: Tues November 18, 2014, 9:30am

Exercise 58 Sentence 1 of Example 3.1.2 can be written as.

$$\forall x(\text{Healthy}(x) \rightarrow \neg\text{Dead}(x)).$$

Translate all other sentences from Example 3.1.2. Use schroedinger as a constant symbol and use only the following predicate symbols:
unary: Healthy, Dead, Cat, Alive, HappyCatOwner
binary: owns, cares

Exercise 59 Sketch how you could formally prove, using the formulas from Exercise 58, that Schrödinger's cat is alive.

Exercise 60 Show, without using any of the statements in Theorem 3.4.1, that the first statement, $\neg\forall xF \equiv \exists x\neg F$, holds.

Exercise 61 Show, that $\forall x\exists yP(x,y) \not\equiv \exists u\forall vP(v,u)$.

Exercise 62 Show, using the statements from Theorem 3.4.1, that $\forall x\exists y(P(x)\wedge Q(y)) \equiv \exists y\forall x(P(x)\wedge Q(y))$.

Exercise 63 Show by using the statements from of Theorem 3.4.1, that

$$\forall x(P(x) \rightarrow (\exists y(O(x,y) \wedge C(y)) \wedge (\forall z(R(x,z) \rightarrow H(z)))))$$

and
$$\forall z\forall x\exists y((P(x) \rightarrow (O(x,y) \wedge C(y))) \wedge ((P(x) \wedge R(x,z)) \rightarrow H(z)))$$

are equivalent.

Exercise 64 What is $(\forall x(Q(x,y,z)[y/a])[x/b] \wedge \forall x(P(x,y)[y/x][x/a]))[z/x]$?