CS 740 – Computational Complexity and Algorithm Analysis

Spring Quarter 2010

Slides 1

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Today's Session

1. A motivating example
2. What is *Computational Complexity* all about?
3. More examples
4. A computational complexity success story
5. Organizational matters

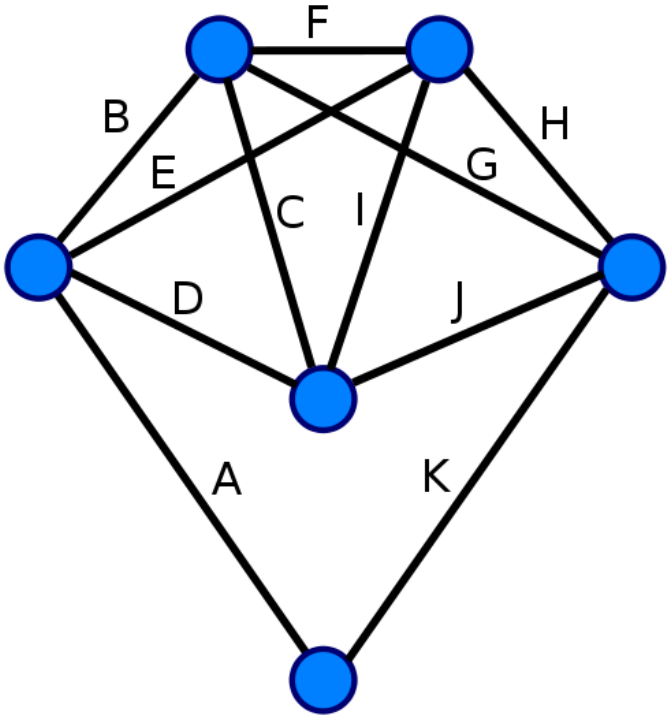
A motivating example

Input: A connected graph (undirected)

Output: “yes” if the graph has a path which

- **visits each edge exactly once and**
- **starts and ends on the same vertex.**

Output: “no” otherwise



Find an algorithm for this problem.

[Such graphs are called *Eulerian*.]

[Such paths are called *Eulerian cycles*.]

Naive algorithm 1 – brute force

- Graph consists of
  - m vertices: 1,2,...,m
  - n edges, written as (x,y) [edge between vertex x and vertex y]

```
for each permutation P of the n edges
      [i.e., P = ( (x1,y1),...,(xn,yn) )]
  output "yes" if P constitutes an Eulerian cycle
output "no" if no Eulerian cycle was found
```

Is this a good algorithm?

How to improve?

Naive algorithm 1 – brute force

- **Graph consists of**
  - **m vertices: 1,2,...,m**
  - **n edges, written as (x,y) [edge between vertex x and vertex y]**

```
for each permutation P of the n edges
      [i.e., P = ( (x1,y1),...,(xn,yn) )]
  output “yes” if P constitutes an Eulerian cycle
output “no” if no Eulerian cycle was found
```

How costly is this? (roughly, order of magnitude)

If no Eulerian cycle is found, we have to check n! permutations (that’s the *worst case*).

n! is quite a lot!

n	n!
5	120
10	3,628,800
15	$\approx 1.3 \cdot 10^{12}$
20	$\approx 2.4 \cdot 10^{18}$
50	$\approx 3 \cdot 10^{64}$
70	$\approx 10^{100}$

10^{100} – That's more than there are particles in the universe

n! is quite a lot!

• 10^1	
• 10^2	
• 10^3	Number of students in the college of engineering
• 10^4	Number of students enrolled at WSU
• 10^6	Number of people in Dayton
• 10^7	Number of people in Ohio Number of seconds in a year
• 10^8	Number of people in Germany
• 10^10	Number of stars in the galaxy Number of people on earth Number of milliseconds per year
• 10^20	Number of stars in the universe
• 10^80	Number of particles in the universe

Naive algorithm 2 - backtracking

- Graph consists of
  - m vertices: 1,2,...,m
  - n edges, written as (x,y) [edge between vertex x and vertex y]

```
Fix the first vertex, say, x₁.
Make a systematic depth-first search on the graph
            edges.
  For each resulting maximal path P, if P is an
            Eulerian cycle, output “yes”.
If no Eulerian cycle is found, output “no”.
```

Algorithm is better – but is it *significantly* better?

In the worst case, fully connected graph, we have to check m! paths.

When do we know we have *the best* algorithm?

Smart algorithm

Theorem:

A connected undirected graph is Eulerian if and only if every vertex has an even number of edges (counting loops twice).

```
For each vertex x
  if number of edges of x is odd, output “No” and
                stop.
Output “Yes”.
```

In the worst case, we have to make m checks, each of which consists of counting at most n edges.

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2. **What is *Computational Complexity* all about?**
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What is Comp. Complexity all about?

- Some problems seem hard but are not. Identify them.
- Some problems seem easy but are not. Identify them.

- Know when to stop searching for a smarter algorithm. [And instead turn to optimizations and heuristics.]

- What does "computationally hard problem" mean exactly?
- In what sense can we really say that some problem is computationally harder than some other problem?

What is Comp. Complexity all about?

- It’s a part of *theoretical* computer science.
- It’s a formal theory of the analysis of computational hardness of problems.
- It’s probably rarely going to help you directly in practice.
- But indirectly, in form of having a systematic understanding of problem hardness, it is indispensable.

What is Comp. Complexity all about?

We will certainly also learn about the

P = NP?

problem.

What it is.
Why it is important.

Why some people make such a fuzz about it.

Some basic notions

- **Problem:**
  A mapping from input to output.

- **Algorithm:**
  A method or a process followed to solve a problem.

- **A problem can have many algorithms.**

Some basic notions

- **Problem:**
  A mapping from input to output.

- **Algorithm:**
  A method or a process followed to solve a problem.

- **We focus on problems. Algorithm analysis is also interesting, but not as foundationally important.**

Some basic notions

- **Problem:**
  A mapping from input to output.
  - **We use the *order of magnitude* of the number of steps needed to solve a problem.**
    - **measured as a number which depends on the *input size*.**
  - **We are really interested in the *worst case* scenario.**
    - **i.e., how many steps do we need if the input is as unfavorable as possible?**

Can also be studied:

best case (usually not that interesting)

average case (of practical interest for concrete algorithms)

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Linear Search

- **Input:** **An array *A* of integers, and a value *v*.**
- **Output:** **“Yes” if *v* is an element of *A*;**
  “No” otherwise.

A =

3	12	7	25	7	32	11	56	28	43	6	87	68	91	2

v = 28

Algorithms: *Exhaustive search; random search; sort and linear search; sort and binary search*

Linear Search

Not all inputs of a given size take the same time to run.

Sequential search for *v* in an array of *n* integers:

- **Begin at first element in array and look at each element in turn until *v* is found**

Best case:

Worst case:

Average case?

Linear Search – Average case

Case: i	Time: T(i)	Probability P(i)	Cost: T(i) * P(i)
1	1	1/n	1/n
2	2	1/n	2/n
3	3	1/n	3/n
…	…	…	…
n	n	1/n	1

$$\sum Cost = \tfrac{1}{n} + \tfrac{2}{n} + \cdots + \tfrac{n}{n}$$

$$= \frac{1}{n} \times \sum_{i=1}^{n} i$$

$$= \frac{1}{n} \times \frac{n(n+1)}{2} = \frac{n+1}{2}$$

Bubble Sort

- **Algorithm 1:**

```
for pass = 0…n-1 {
  for position = 0…n-1 {
    if (array[position] > array[position+1]) {
      swap (array[position], array[position+1])
    }
  }
}
```

Best, worst, average: $\approx (n^2)$

Bubble Sort

- **Algorithm 2:**

```
for pass = 0…n-1 {
  for position = 0…n-pass-1 {
    if (array[position] > array[position+1]) {
      swap (array[position], array[position+1])
    }
  }
}
```

Best, worst, average: $\approx (n^2)$

(Within a constant factor)

Bubble Sort

- **Algorithm 3:**

```
for pass = 0…n-1 {
  swaps = 0;
  for position = 0…n-pass-1 {
    if (array[position] > array[position+1]) {
      swap (array[position], array[position+1]);
      swaps++;
    }
  if (swaps == 0) return;
  }
}
```

Best: $\approx n$,

Worst: $\approx (n^2)$

Average = ??

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A success story

- Research Area: Semantic Web
  Aimed at endowing information on the World Wide Web with "machine-processable meaning" (semantics).

- This is done using languages for representing knowledge.
  E.g., the knowledge on a website.
- These languages can also be used for querying this knowledge.

- These languages are also able to represent problems.
  Knowledge: A graph.
  Query: Does it have an Eulerian cycle?

- These languages differ in how "complex" the problems representable in them can be.

A success story

- **Web Ontology Language OWL**
  Recommended standard by the World Wide Web Consortium W3C.
  Established 2004, revised 2009.

- **Research which led to OWL was driven by computational complexity analysis.**

- **Complexity used as a priori measure for runtime.**

- **Goal was finding a language which allows maximum freedom in specifying knowledge (problems), while being of minimal complexity.**

- **This approach paid of extremely well:**
  Currently e.g. substantial commercial interest generated.

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Organizational Matters

- **Office Hours: Wed 3-4, Joshi 389.**
  Email contact preferred.

- **Textbook (required):**
  Thomas A. Sudkamp, Languages and Machines, Third Edition, Addison Wesley, 2006.

- **Textbook (supplementary):**
  Michael R. Garey and David S. Johnson, Computers and Intractability, Freeman, 1979

- **Grading:**
  Midterm exam: 30%
  Final exam: 50%
  Exercises: 20%

Organizational matters

- **I will be absent May 3rd and 5th.**
  - **How do we make up for this?**
    - **Have 7.5 sessions 20 minutes longer (agreed).**
    - **Find alternative dates for meeting.**
    - **Give you extra hand-in homework.**

- **We will frequently make exercise sessions.**
  You will get exercises, to be done at home and graded by me, and discussed afterwards in class.
  Each exercise usually counts 4 points [exceptions are marked].
  Exercises are due *one week after I pose them*.

- **Webpage/slides.**
  I prefer to use a public website:
  http://knoesis.wright.edu/faculty/pascal/teaching/s10/complexity.html

Course overview

Tentative

We recap most of chapter 8

We cover most of chapters 14 and 15

We cover parts of chapters 16 and 17, tbd what/how much.

- **March 29/31: Introduction. Big-O notation.**
- **April 5/7: Recap Turing Machines. Exercise session.**
- **April 12/14: Time complexity.**
- **April 19/21:**
- **April 26/28: mid-term exam**
- **May 3/5: no classes**
- **May 10/12:**
- **May 17/19:**
- **May 24/26:**
- **June 2:**